Development of a Polarized Positron Source by laser Compton Scattering using an Optical Resonant Cavity

博士論文

レーザー蓄積共振器を用いた
レーザーコンプトン散乱による偏極陽電子源開発

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Photon Generation by Laser-Compton Scattering Using an Optical Resonant Cavity at the KEK-ATF Electron Ring
主論文
Abstract

We studied a gamma ray generation system based on laser Compton scattering for the ILC polarized positron source. To increase the intensity of polarized positrons, we adopted a scheme to enhance laser intensity by an optical resonant cavity.

For the purpose of the development of laser Compton scheme with the optical resonant cavity, we installed an optical resonant cavity to the KEK-ATF electron beam damping ring. It was confirmed that the installation of the optical resonant cavity did not affect the operation of the ATF which required high vacuum to generate low emittance electron beams. We successfully stacked mode locked laser pulses and enhanced intensity of laser pulses to 760 times of injected intensity in the optical resonant cavity. Stacked laser pulses were collided with multi bunch electron beams. When stacked laser power was $1.04 \pm 0.02$ kW and electron current was 9.0 mA, number of detected gamma rays were $53.9 \pm 1.1$ /train with the electron beam of 10 bunches/train. It is corresponded to $(3.53 \pm 0.07) \times 10^8$ gamma rays /second in all solid angles. It was also confirmed that number of gamma rays were consistent with estimation by a numerical simulation.

For future study toward the ILC positron source, we constructed a new detector system to measure bunch by bunch gamma ray yield and preliminary results were obtained.
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Chapter 1

Introduction

1.1 Generation method of high energy photon

Röntgen discovered X-rays in 1895 [1]. In Röntgen’s experiment, X-rays were generated by accelerated electrons in a Crookes tube. Even now, high energy photons are generated by using high energy electron beams. For example, several keV photon is generated by the transition of energy level in an atom which are excited by bombarding electrons. When the electron in a higher energy level falls to a low energy level in the atom, a photon of energy of difference between levels is emitted. It is the characteristic X-ray. In the copper atom, the energy of characteristic X-ray, when a electron falls into the most inner level (k shell), is about 8 keV.

The other method of generating high energy photons is to strike high energy electrons to a material. When high energy electrons impinge into the material, the electrons lose its energy by collision with atoms in the material and emit as photons. It is called the braking radiation or the bremsstrahlung. The maximum energy of the bremsstrahlung reaches energy of the incident electron but its spectrum is widely spread from almost zero to its maximum energy. As an other method of generating photons, synchrotron radiation is used. Synchrotron radiation is a phenomenon that charged particles emit photons when it was bent by the Lorentz force. Fig. 1.1 shows energy spectrum of generated photon in SPring-8 [2]. The maximum energy is about 300 keV with the 8 GeV electron beam.

Laser Compton scattering it is an another way to generate photons by the scattering of the laser photon off the high energy electron. An advantage of laser Compton scheme is that the higher energy photon can be generated with the relatively lower energy electron. For example, $O(10\ \text{MeV})$ photon can be generated with the $O(1\ \text{GeV})$ electron, which
contrast with the synchrotron radiation (300 keV photon with 8 GeV electron). In addition, polarized photons can be generated and easily controlled in laser Compton scattering. For this reason, the laser Compton scattering is attractive as compact, high energy, polarized photon sources from X-rays to high energy gamma rays.

A possible application of laser Compton scattering is a polarized electron source for the International Linear Collider (ILC). The ILC is an electron position linear collider to explore physics in Tera scale as will be described in detail in §1.3. We have been working to develop a polarized positron source for the ILC using polarized gamma rays generated by laser Compton scattering. Laser Compton scattering has advantage for generating polarized positrons with lower electron energy. However, it is a price to pay, i.e., we need technical developments to generate sufficient number of positrons required by the ILC. In this article, we describe status and prospect of the research and development for the polarized positron source using the laser Compton scattering.

Figure 1.1: Synchrotron Radiation Spectrum of SPring-8 [2].

Figure 1.2: Total cross section of the photon irradiated into a carbon and a lead [3] pp. 264.
1.2 Polarized electron and positron

1.2.1 Method of positron generation

Positrons are generated in various methods. They can be obtained from $\beta^+$ decay of $^{22}$Na and $^{68}$Ge \cite{3}, pp. 296. However, $\beta^+$ decay happens at random in time due to stochastic nature of radioactive decay and is difficult to implement it to a positron source for accelerators.

The other and widely used method for accelerators is to use the electromagnetic shower in metal targets such as a lead or a tungsten. In this article, it is called the electron driven scheme. In the electron driven scheme, the incident electron generates photons by bremsstrahlung in a target. Generated photon causes pair creation to electrons and positrons in the target, then electrons and positrons generate photons by bremsstrahlung again. These reactions happen repeatedly and form a cascade shower in the target. Finally, positrons are collected by a capture system consists of magnets and accelerating structure which follows the target and sent further acceleration system.

Similarly as the electron driving scheme, when the high energy photon is injected to the target, positrons are also created via the cascade shower. The time structure of the positron generation can be controlled by controlling the timing of incident particles. Fig. 1 shows total cross section of the photon irradiated into a carbon and a lead. $\sigma_{\text{p.e.}}$ is a atomic photoelectron effect. $\kappa_{\text{nuc}}$ is a pair creation in the field of the nuclear. $\kappa_e$ is a pair creation in the field of the electron. In the lead target, the pair creation process is dominant above photon energy of $O(10 \text{ MeV})$. Therefore, either the electron driven of the photon driven, one need photon of $O(10 \text{ MeV})$ or more for effective generation of positrons in the target.

1.2.2 Physical significance of polarized beam

The Large Electron-Positron Collider (LEP) and the SLAC Linear Collider (SLC) were constructed for the experimental verification of the standard theory up to 200 GeV energy range. The LEP was in operation from 1989 to 2000 at CERN (European Organization for Nuclear Research) and the SLC was operated from 1989 to 1999 at SLAC (Stanford Linear Accelerator Center \footnote{Currently SLAC National Accelerator Laboratory}) \cite{4}. Diagrams of the lowest order $e^+e^-$ annihilation in the Standard Model are shown in Fig. 1.
In figure, $e^-$, $e^+$, $f$, $\bar{f}$, $\gamma$, $Z$, $h$, $W^\pm$ and $\nu$ are, an electron, positron, fermion, anti-fermion, photon, Z boson, Higgs boson, W boson and neutrino, respectively. R and L denote the helicity of the electron and the positron for the right handed and the left handed ones, respectively. Among these the interaction, (f) in Fig. 1.3 is highly suppressed and (g) is prohibited by the SU(2)$_L \times$ U(1) nature of the Standard Model [6, 7]. The electron of different helicity behaves as the different particle from the interaction point of view in the Standard Model. Therefore, the polarized electron beam, which is to choose the helicity of initial state, is useful to investigate property of the interaction in the Standard Model.

The polarized electron beam is used in SLC while the unpolarized electron beam is used in LEP. SLC had 1 detector as SLD. LEP had 4 detectors, ALEPH, DELPHI, L3, OPAL. The weak mixing angle, which is one of the key parameter of the Standard Model, was measured both in the SLC and the LEP experiments as follows:

$$\sin^2 \theta_W = 0.2310 \pm 0.00054 \pm 0.00014 \quad \text{(SLD/SLC)}$$

$$\sin^2 \theta_W = 0.2322 \pm 0.0008 \pm 0.0011 \quad \text{(ALEPH/LEP)}$$

First error is the statistical error and second error is the systematic error. The systematic error of the SLD/SLC is one order smaller than the ALEPH/LEP, while the statistical error is comparable level. However, the integrated luminosity is factor 20 larger for the ALEPH/LEP experiment than the SLD/SLC. This showed that the polarized beam is powerful tool for physics with the electron position interactions.

In addition to using the polarized electron beam, if we use the polarized
1.2. POLARIZED ELECTRON AND POSITRON

The effective polarization $P_{\text{eff}}$ can be:

$$P_{\text{eff}} = \frac{P_e - P_p}{1 - P_e P_p},$$

where $P_e$ is the polarization of the electron beam and $P_p$ is the polarization of the positron beam. $P = 1$ means the polarization of the beam is 100 % right handed while $P = -1$ means the beam is 100 % left handed. For example, when the polarization of the electron beam is 80 % ($P_e=0.8$) and the polarization of the positron beam is 60 % ($P_p=-0.6$). The effective polarization is about 95 %. Thus, high precision measurements are expected using the polarized electron and positron beam. Fig. 1.4 shows the cross section of the electron and the positron to weak boson pairs ((e), (f), (g) and (h) in Fig. 1.3). The horizontal axis is the electron-positron collision energy. The red line is the case that both electron and positron beams are unpolarized. The blue line is the case that the electron beam is 80 % polarized and the positron beam is unpolarized. The green line is the case that the polarization of the electron beam is 80 % and that of the positron is -60 %. When only the electron is polarized, the cross section becomes $1/5$ of unpolarized case. When both electrons and positrons are polarized, cross section becomes $1/10$ of unpolarized case. For case that W boson pairs are background for desired physics analysis, the W boson can be suppressed factor 10 by using the polarized electron and positron beam.

1.2.3 Method of polarized beam generation

Polarized electrons can be generated by a DC gun of GaAs photocathode [12, 13]. The electron beam of 90 % polarization was achieved by using the GaAs-GaAsP strained superlattice layers photocathode [14].

On the other hand, polarized positrons are generated using polarized photons. Fig. 1.5 shows the principle of the polarized positron generation by a polarized gamma ray [15]. A polarized gamma ray is injected to a thin target and a positron is created via a pair creation process. At this time, a positron and an electron are polarized thanks to the helicity conservation. When the gamma ray is left handed, the polarizations of the electron and the positron is left handed in the high energy end of the positron.
Figure 1.4: Cross section from the electron and the positron to weak boson pairs \([11]\). The red line is the case the both the electron and the positron beam are unpolarized. The blue line is the case that the electron beam is 80% polarized and the positron beam is unpolarized. The green line is the case that the polarization of the electron beam is 80% and that of the positron is \(-60\%\).

Figure 1.5: The principle of the polarized positron generation by the polarized gamma ray.

1.3 The International Linear Collider

1.3.1 Targets of the ILC

The International Linear Collider (ILC) is an electron positron linear collider with the center of mass energy of \(\sim 200\) GeV to 1 TeV. The aim of the ILC it explore physics at Tera scale such as \([12]\):

- Precision measurement of properties of the Higgs particle which is expected to be discovered at the LHC.
- Search for physics beyond the Standard Model, such as Super Symmetry.
- Search for a signature of new physics via precise measurements of the Standard Model parameters.
1.3. THE INTERNATIONAL LINEAR COLLIDER

Physics case for the ILC is extensively studied and it is believed that studies at the ILC with combination with the LHC will play a key role to explore physics at Tera scale and possibly more [17].

1.3.2 Limitation of circular accelerators

When electrons or positrons are accelerated in a circular accelerator, they lose their energy by the synchrotron radiation. The energy loss is significant at high energy and is a serious problem for high energy accelerators. For a charged particle of mass $m$ and energy $E$, bent with the curvature radius $\rho$, the energy loss by the synchrotron radiation per one turn is:

$$\delta E = \frac{4\pi}{3} \frac{e^2}{(mc^2)^2} \frac{E^4}{\rho}.$$  

- $E$: energy of particle,
- $\rho$: curvature radius of circular accelerator,
- $m$: mass of particle,
- $c$: light velocity.

For instance, $E = 500$ GeV, $\rho = 4.3$ km, the energy loss is about 1.3 TeV/turn [12]. $\rho = 4.3$ km is the curvature radius of the LEP which is the world largest accelerator at this moment. It means a circular accelerator is by far a practical solution for accelerating the electron and the positron of such high energy.

1.3.3 Configuration of the ILC

A planned accelerator complex of the ILC is shown in Fig. [18]. Principal parameters of ILC are summarized in Table [14].

![Figure 1.6: A planned accelerator complex of the International Linear Collider [15].](image)
Table 1.1: Basic design parameters of ILC

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam energy</td>
<td>250 - 500 GeV</td>
</tr>
<tr>
<td>Number of bunches</td>
<td>1000 - 5400</td>
</tr>
<tr>
<td>Bunch spacing</td>
<td>369 ns</td>
</tr>
<tr>
<td>Bunch charge</td>
<td>1.6 - 3.2 nC</td>
</tr>
<tr>
<td>Accelerating gradient</td>
<td>31.5 MV/m</td>
</tr>
<tr>
<td>Normalized horizontal emittance</td>
<td>10 mm. mirad</td>
</tr>
<tr>
<td>Normalized vertical emittance</td>
<td>0.04 mm. mirad</td>
</tr>
<tr>
<td>Bunch length</td>
<td>0.3 mm</td>
</tr>
<tr>
<td>Horizontal beam size</td>
<td>640 nm</td>
</tr>
<tr>
<td>Vertical beam size</td>
<td>5.7 nm</td>
</tr>
<tr>
<td>Luminosity</td>
<td>$2.00 \times 10^{34}$ cm$^{-2}$ s$^{-1}$</td>
</tr>
</tbody>
</table>

In the circular accelerator, beams go around in the ring and accelerated during the circulation. Also, electrons and positrons circulating opposite direction keep intersecting each other during their life time in the accelerator. On the other hand, in the linear accelerator, electrons and positrons are accelerated and get collided only once when they travel through the accelerator. Therefore, the technology that accelerates beams to sufficient energy and makes effective collision are needed. One of the key component is to accelerating units with high accelerating gradient. As for the accelerated gradient, 40.7 MV/m is achieved at KEK (Japan) on December 2010 [19]. The other issue is to realize small beam size at the electron positron interaction point to increase luminosity. For this purpose, generation of a low emittance beam and a focusing system to squeeze beam down to the small size is crucial. Creation of the low emittance beam and the focusing system for the ILC are studied at the KEK-ATF as explained in §3.1.

1.3.4 Method of polarized positron generation at the ILC

The requirements for the positron source for the ILC are shown in Table 1.2. The helical undulator scheme is considered as the base line design. The electron driven scheme and the laser Compton scheme are studied for the polarized positron source too.
1.3. **THE INTERNATIONAL LINEAR COLLIDER**

Table 1.2: Positron parameters of ILC [12].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positron per bunch at the IP</td>
<td>2 x 10^{10}</td>
</tr>
<tr>
<td>Number of bunches</td>
<td>2625</td>
</tr>
<tr>
<td>Pulse repetition rate</td>
<td>5 Hz (125 to 250 GeV)</td>
</tr>
</tbody>
</table>

1.3.5 **Method of polarized gamma ray generation**

The gamma rays of 10 MeV or more are necessary for efficient pair creations as discussed in the previous section. A possible scheme to meet the requirements is using a helical undulator. The incident electron moves meander shape in the undulator as shown Fig. 1.7. In the helical undulator, the incident electron moves spirally in the helical coil. Fig. 1.8 shows double helical coil of the helical undulator.

![Figure 1.7: A schematic layout of the undulator.](image1.png)

![Figure 1.8: A schematic layout of the helical undulator [20].](image2.png)

When the electron moves spiral trajectories by the magnetic field, the energy of generated polarized photon is [21];

$$E_u = \frac{2}{1 + K^2 \gamma^2} \hbar \omega_0.$$  

*K* value of the undulator is defined as [21];

$$K = \frac{e B_0 \lambda_u}{2 \pi m_0 c}.$$  

Here \(\lambda_u\) is the undulator period. \(B_0\) is magnetic field of center axis. \(\omega_0\) shows orbital rotation angular frequency of the spiral motion.

There are issues for the undulator scheme; the electron beam of more than 150 GeV is needed to generate the polarized gamma rays of 10 MeV or more, therefore, the electron beam accelerated by main linac is used for the undulator. It means that the positron beam cannot be provided until the electron beam side start to operate.
The other method to create photons of $O(10 \text{ MeV})$ is to use laser Compton scattering. Laser Compton scattering is collision of the high energy electron beam with the laser beam. Fig. 1.9 is a schematic of laser Compton scattering. When the incident laser is polarized in right handed state, the scattered photon is left handed at high energy end.

When the laser collides head on with the electron beam, the maximum photon energy is approximately expressed as;

$$E_C \sim 4\gamma^2 E_0,$$

$E_C$ is the energy of generated photon to the forward direction of the electron beam. $E_0$ is the energy of the laser beam.

Laser Compton scheme generates gamma rays of 10 MeV or more with several GeV electrons. It is possible to develop the positron source independently from the electron beam. In addition, polarization of the generation gamma rays can be controlled easily by controlling the polarization of the laser. However, the cross section of laser Compton scattering is small, and technical developments are needed to increase number of gamma rays generation.

1.3.6 Research and development of laser Compton scheme

Polarized gamma ray generation by laser Compton scheme was verified by previous experiment [22]. The experiment for polarized positron generation was conducted too [23]. $2 \times 10^4$ polarized positrons / bunch were generated in the experiment, and measured polarization was $73 \pm 15$ (statistical error) $\pm 19$ (systematic error) % [23]. The next step toward the polarized positron source is to increase the intensity of the electron beam and the laser beam more to match the demand of the ILC.

Several schemes are studied to achieve the number of generation for ILC by the laser Compton scattering [24, 25]. One scheme is a Compton ring scheme. The Compton ring scheme is a method of accumulating the electron beam in the storage ring and the multiple laser Compton
interaction points are constructed in the ring to increase the photon intensity. Fig. 1.10 shows a schematic layout of the Compton ring scheme. Designed parameters for a Compton ring are summarized in Table 1.3. Energy of the electron beam is 1.8 GeV. The laser pulses accumulate for increasing intensity in the optical resonant cavity which is like a coupled mirror. Five optical resonant cavities are placed, and intensity will be 0.6 J each interaction point. $2 \times 10^{10}$ positrons will be generated with colliding every 10 msec.

<table>
<thead>
<tr>
<th>Electron beam</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam energy</td>
<td>1.8 GeV</td>
</tr>
<tr>
<td>Bunch spacing</td>
<td>6.2 ns</td>
</tr>
<tr>
<td>Bunch intensity</td>
<td>5.3 nC</td>
</tr>
<tr>
<td>Pulse repetition rate</td>
<td>100 Hz</td>
</tr>
<tr>
<td>Number of bunches</td>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Laser beam</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pulse intensity</td>
<td>0.6 J</td>
</tr>
<tr>
<td>Number of optical resonant cavities</td>
<td>5</td>
</tr>
</tbody>
</table>

1.4 About this work

1.4.1 Objective of the study in the KEK-ATF damping ring

In this work, we aim to increase number of gamma rays and repetition rate by the Compton ring scheme. A key issue is a development of an optical resonant cavity for the Compton ring scheme. It includes overall conditions in the accelerator environment: laser optics; synchronization with laser pulses and electron beams; resonance control; suppression of
electrical noise; and vacuum system. We installed a laser Compton system consists of an optical resonant cavity and a mode locked pulsed laser in the KEK-ATF damping ring. The electron energy of the KEK-ATF is 1.3 GeV. The bunch spacing is 2.8 ns (or 5.6 ns). They are close to one we need for the Compton ring scheme for the ILC. Our aims are high intensity gamma ray generation of high repetition rate with the optical resonant cavity in the KEK-ATF damping ring.

In addition to parameters of the electron beam, other research programs that use the laser and the optical resonant cavity are being performed at the KEK-ATF: laser wire beam profile monitor at damping ring [27]; laser wire beam profile monitor at extraction line [28]; and laser Undulator Compact X-ray source facility [29] are studying in the KEK-ATF. The KEK-ATF is the best facility and environment for studying Compton ring scheme with the optical resonant cavity for technical as well as collaborative aspects.

1.4.2 About this article

In chapter 2, theoretical background of work; energy distribution of scattered photon; laser pulse stacking to the optical resonant cavity; and techniques of cavity length control for keeping resonance of the optical resonant cavity were described.

Chapter 3 and chapter 4 are devoted to the description of the setup and results for gamma ray generation experiment.

The setup and results for bunch-by-bunch measurement after increasing enhancement factor of the optical resonant cavity are described in Chapter 5 and chapter 6.

Chapter 7 is the conclusion of this article.
Chapter 2

Theoretical Description of Compton Scattering and Laser Pulse Stacking

2.1 Compton scattering

2.1.1 Scattered photon energy

Four-momentums of electrons and photons in the initial electron rest frame are described as follows;

\[ P^{\gamma}_{\gamma_1} = \frac{E^{\gamma}_1}{c} \begin{pmatrix} c \\ \cos \theta^* \\ \sin \theta^* \\ 0 \end{pmatrix}, \quad P^{e_1} = \frac{mc^2}{c} \begin{pmatrix} c \\ 0 \\ 0 \\ 0 \end{pmatrix}, \]

\[ P^{\gamma}_{\gamma_2} = \frac{E^{\gamma}_2}{c} \begin{pmatrix} c \\ \cos \phi^* \\ \sin \phi^* \\ 0 \end{pmatrix}, \quad P^{e_2} = \frac{mc^2 \gamma^*}{c} \begin{pmatrix} c \\ \beta^* \cos \psi^* \\ \beta^* \sin \psi^* \\ 0 \end{pmatrix}, \]

where \( E^{\gamma}_1 \) is the energy of the incident photon, \( E^{\gamma}_2 \) is the energy of the scattered photon, \( \theta^* \) is the collision angle and \( \phi^* \) is the scattered angle of the photon as shown in Fig. [Fig.]

From the energy momentum conservation;

\[ P^{\gamma}_{\gamma_1} + P^{e_1} = P^{\gamma}_{\gamma_2} + P^{e_2}, \]

\[ \left| P^{\gamma}_{\gamma_2} \right|^2 = \left| P^{\gamma}_{\gamma_1} + P^{e_1} - P^{\gamma}_{\gamma_2} \right|^2 = \right| P^{\gamma}_{\gamma_1} P^{\gamma}_{\gamma_1} + P^{e_1} P^{e_1} + P^{\gamma}_{\gamma_2} P^{\gamma}_{\gamma_2} + 2 P^{\gamma}_{\gamma_1} P^{e_1} + 2 P^{\gamma}_{\gamma_2} P^{e_1} - 2 P^{\gamma}_{\gamma_1} P^{e_1} - 2 P^{\gamma}_{\gamma_1} P^{e_1} \]

(2.1)
CHAPTER 2. THEORETICAL DESCRIPTION OF COMPTON SCATTERING AND LASER PULSE STACKING

Figure 2.1: Schematic of Compton scattering in the rest frame of the initial electron.

Figure 2.2: Schematic of Compton scattering in the laboratory frame.

the left side of the Eq. (2.2) is;

\[-m^2c^2\gamma^*^2 + m^2c^2\beta^*^2\gamma^*^2(\sin^2\psi^*^2 + \cos^2\psi^*^2)\]

\[= -m^2c^2, \quad (2.2)\]

the right side is;

\[0 - m^2c^2 + 0 - 2mE_1^* + 2mE_2^* + 2\frac{E_1^*E_2^*}{c^2}(1 - \cos\theta^*\cos\phi^* - \sin\theta^*\sin\phi^*)\]

\[= -m^2c^2 - 2mcE_1^* + 2mcE_2^* + 2\frac{E_1^*E_2^*}{c^2}[1 - \cos(\theta^* - \phi^*)]. \quad (2.3)\]

From Eq. (2.2) and Eq. (2.3), the energy of the scattered photon in the initial electron rest frame; \(E_2^*\) is

\[E_2^* = \frac{mc^2E_1^*}{mc^2 + E_1^*[1 - \cos(\theta^* - \phi^*)]}\].

The kinematic variables in the rest frame can be converted to the ones
in the laboratory frame by Lorentz transformation as:

\[ E_1^* = \gamma E_1 (1 - \beta \cos \theta), \]
\[ E_2^* = \gamma E_2 (1 - \beta \cos \phi), \]
\[ \theta^* = \left\{ \begin{array}{l}
\sin^{-1} \left[ \frac{\sin \theta}{\gamma (1 - \cos \theta)} \right], \\
\cos^{-1} \left( \frac{\beta + \cos \theta}{1 - \cos \theta} \right), \\
\sin^{-1} \left[ \frac{\sin \phi}{\gamma (1 - \cos \phi)} \right], \\
\cos^{-1} \left( \frac{\beta + \cos \phi}{1 - \cos \phi} \right),
\end{array} \right. \]
\[ \phi^* = \left\{ \begin{array}{l}
\sin^{-1} \left[ \frac{\sin \theta}{\gamma (1 - \cos \theta)} \right], \\
\cos^{-1} \left( \frac{\beta + \cos \theta}{1 - \cos \theta} \right), \\
\sin^{-1} \left[ \frac{\sin \phi}{\gamma (1 - \cos \phi)} \right], \\
\cos^{-1} \left( \frac{\beta + \cos \phi}{1 - \cos \phi} \right),
\end{array} \right. \]

where \( E_1 \) is the energy of the incident photon, \( E_2 \) is the energy of the scattered photon, \( \theta \) is the collision angle and \( \phi \) is the scattered angle of the photon as shown in Fig. 2.2. The scattered photon energy in the laboratory frame, \( E_2 \), is expressed as:

\[ E_2 = \frac{mc^2 \gamma E_1 (1 - \beta \cos \theta)}{mc^2 \gamma (1 - \beta \cos \phi) + E_1 [1 - \cos (\theta - \phi)]}. \]

For the case that a laser photon collides head on (\( \theta = 180 \text{ deg} \)) with a high energy electron, the photon of the maximum energy is scattered to forward (\( \phi = 0 \text{ deg} \)) region with the energy of

\[ E_{2\text{max}} = \frac{\gamma^2 E_1 (1 + \beta)^2}{1 + \frac{4 \gamma E_1}{mc^2} (1 + \beta)} \sim 4 \gamma^2 E_1. \]

Here \( \beta \sim 1 \) and \( \frac{4 \gamma E_1}{mc^2} (1 + \beta) \ll 1 \) because we assume that \( E_1 \sim 1 \text{ eV} \) as the laser light and \( \gamma \sim 2000 \) as the electron energy \( \sim 1 \text{ GeV} \). For \( mc^2 \gamma = 1.28 \text{ GeV} \), \( E_1 = 1.165 \text{ eV} \), maximum energy of scattered photon is about 29 MeV. While \( mc^2 \gamma = 1.28 \text{ GeV} \), \( E_1 = 1.165 \text{ eV} \) and \( \theta = 192 \text{ deg} \), which is our experimental condition described later, the maximum photon energy is 28.4 MeV at \( \phi \simeq 0 \text{ deg} \). Fig. 2.3 shows the scattered photon energy versus the scattered angle with experimental condition.

### 2.1.2 Differential cross section

The differential cross section of Compton scattering in the electron rest frame is given by [30]:
Figure 2.3: Energy of scattered photon as a function of scattering angle. Incident electron energy, laser photon energy and collision angle are assumed as 1.28 GeV, 1.165 eV and 192 deg, respectively.

\[
\frac{d\sigma}{d\Omega^*} = \frac{r_0^2}{2} \left( \frac{E_2^*}{E_1^*} \right)^2 \left[ \frac{E_1^*}{E_2^*} + \frac{E_2^*}{E_1^*} - \sin^2 (\phi^* - \theta^*) \right].
\]

The expression can be converted to the laboratory frame as;

\[
\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega^*} \frac{d\Omega^*}{d\Omega} = \frac{d\sigma}{d\Omega^*} \frac{\sin \phi^*}{\sin \phi} \frac{d\phi^*}{d\phi} = \frac{d\sigma}{d\Omega^*} \frac{-1}{\sin \phi} \frac{d}{d\phi} \cos \phi^*.
\]

Fig. 2.4 shows the differential cross section with the same parameters used in Fig. 2.3. Fig. 2.3 shows the energy distribution of the scattered photon in the laboratory frame which is calculated as;

\[
\frac{d\sigma}{dE_2} = \frac{d\sigma}{d\Omega^*} \frac{d\Omega^*}{dE_2}.
\]

Fig. 2.4 shows that more than 30 % gamma rays are generated in 2\(\phi = 0.5 \text{ mr} \), when \(mc^2\gamma = 1.28 \text{ GeV} \), \(E_1 = 1.165 \text{ eV} \), \(\theta = 192 \text{ deg} \). By Fig. 2.3 and Fig. 2.4, the photon of quasi-monochromatic energy can be chosen by collimating angle of the scattered photons.

2.1.3 Differential cross section of polarized Compton scattering

We show differential cross section of polarized Compton scattering. The differential cross section in the laboratory is given by [31];

\[
\frac{1}{\sigma_c} \frac{d\sigma}{d\omega} = \frac{1}{E_0 \sigma_c} \frac{2\pi \alpha^2}{x m^2} \left[ \frac{1}{1 - y} + 1 - y - 4r(1 - r) - \lambda e\lambda \gamma r x (2r - 1)(2 - y) \right],
\]
2.1. COMPTON SCATTERING

![Differential cross section vs. scattered angle](image1)

Figure 2.4: Scattered angle dependence of differential cross section. Incident electron energy, laser photon energy and collision angle are assumed as 1.28 GeV, 1.165 eV and 192 deg, respectively.

![Differential cross section vs. scattered energy](image2)

Figure 2.5: Scattered energy dependence of differential cross section. Incident electron energy, laser photon energy and collision angle are assumed as 1.28 GeV, 1.165 eV and 192 deg, respectively.

where

\[ x = \frac{4E_0\omega_0}{m^2}\cos^2\left(\frac{\alpha}{2}\right), \]

and

\[ \sigma_c = \sigma_c^0 + \lambda_c\lambda_c^I\sigma_c^I, \]

\[ \sigma_c^0 = \frac{\pi\alpha^2}{2m^2}\left[\left(2 - \frac{8}{x} - \frac{16}{x^2}\right)\ln(x + 1) + 1 + \frac{16}{x} - \frac{1}{(x + 1)^2}\right], \]

\[ \sigma_c^I = \frac{\pi\alpha^2}{2m^2}\left[\left(2 + \frac{4}{x}\right)\ln(x + 1) - 5 + \frac{2}{x + 1} - \frac{1}{(x + 1)^2}\right], \]
where $\lambda_1$ is circular polarization of the incident photon, $\lambda_c$ is longitudinal polarization of the incident electron, $\alpha$ is the angle between the incident electron and photon, $E_b$ is the energy of the incident electron, $\omega_0$ is the frequency of the incident photon, $\omega$ is the frequency of the scattered photon. $y$ and $r$ are defined as:

\[
\begin{align*}
y & = \frac{\omega}{E_b}, \\
r & = \frac{y}{x(y - 1)}.
\end{align*}
\]

Fig. 2.6 shows the energy distribution of the scattered photon when $E_b = 1.28$ GeV, $\hbar \omega_0 = 1.165$ eV, $\alpha = 12$ deg and the polarization of the incident photon is right handed. It is observed that polarized photon can be obtained by discriminating their energy.

![Figure 2.6: Differential cross section on each circular polarizations](image)

2.2 Principle of Laser

2.2.1 Principle of laser oscillation

LASER is the abbreviation of Light Amplification by Stimulated Emission of Radiation. Fig. 2.7 shows an atomic level with energy $E_1$ and $E_2$. The atom is excited from its ground level to $E_2$ when the light of its energy $\hbar \omega = E_2$ is injected and may be de-excited to $E_1$ with a probability by emitting the photon of energy $\hbar \omega_{12}$. It is called the spontaneous emission.

If the atom is illuminated with the light of energy $\hbar \omega_{12}$ while it is excited to $E_2$, it is stimulated by the light and emits a light of energy $\hbar \omega_{12}$. The characteristic of emitted light, such as the wavelength, phase
and polarization, is the same as the characteristic of the injected photon. This emission is called the stimulated emission. In this example, the spontaneous emission as well as the stimulated emission from the $E_2$ to $E_1$ can be a source for the stimulated emission.

Let $N_2$ be the number of atoms excited at $E_2$ and $N_1$ be the number of atoms at $E_1$ in a medium. When the incident light is sufficiently strong so that excitation rate to $E_2$ is larger than the de-excitation rate to $E_1$, the population inversion, $N_2 > N_1$, can be realized. When the medium is in the population inversion, the intensity of stimulated light emission is

$$I_{\text{stimulate}} \propto \exp (N_2 - N_1) I_{\text{incident}},$$

where $I_{\text{incident}}$ is the intensity of the incident light. Therefore, a large number of stimulated emission can be induced by a low intensity light with the population inversion. As mentioned, both spontaneous and stimulated emission can be a source for new stimulated emission so that the cascade of stimulated emissions happens in the medium.

For the laser oscillation, we consider the stimulated emissions in the medium placed in an optical cavity. The optical cavity is a pair of mirrors in which the emitted light in the medium can be confined. If the separation of mirrors satisfies a boundary condition of the emitted light, the intensity of the light is stored and amplified in the optical cavity. For satisfying the boundary condition, the separation of mirrors, the optical cavity length $L_{\text{laser}}$ is:

$$L_{\text{laser}} = n \frac{\lambda}{2} = n \frac{\pi c}{\omega}. \quad (n : \text{natural number}) \quad (2.4)$$

When reflection of mirrors is $R$, the emitted light leaks from the optical cavity at the rate of $1 - R$. If the intensity of light comes out from the cavity and amplification in the cavity is balanced, the system is in oscillation and the continuous light emission is observed. It is called the Continuous Wave (CW) laser. Characteristics of the profile of the laser will be discussed in §

![Figure 2.7: The optical transition of two level system.](image-url)
The frequency of the stimulated emission is not completely the same due to vibrations of atoms and the uncertainty relationship. When the angular frequency of the resonance is $\omega_0$, the laser intensity distribution is approximated as follows [33];

$$|E(\omega)|^2 \propto \frac{1}{(\omega - \omega_0)^2 + (\delta \omega)^2}. $$

$\delta \omega$ is the line width and is defined by the Full Width at Half Maximum (FWHM) of the intensity distribution. If $\delta \omega$ is much smaller than $\omega_0$, which satisfies Eq. (2.4) as shown in Fig. 2.8, it can be regarded as a single frequency oscillation.

![Figure 2.8: This figure shows output frequency distribution from optical cavity when $\delta \omega$ is small.](image)

### 2.2.2 Gaussian beam

We consider the propagation of light which has Gaussian shape transverse intensity like;

$$f(x) = A \exp \left( -\frac{x^2}{2\sigma^2} \right).$$

The electromagnetic wave traveling in the vacuum satisfies the wave equation as;

$$\Delta E(r, t) = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} E(r, t). \quad (2.5)$$

A solution of Eq. (2.5) can be expressed by a form of variable separation as;

$$E(r, t) = \mathcal{E}(r) e^{i\omega t}. \quad (2.6)$$

The spatial part of the solution, $\mathcal{E}(r)$, satisfies Helmholtz equation;

$$\Delta \mathcal{E}(r) + k^2 \mathcal{E}(r) = 0, \quad (2.7)$$
with

\[ k^2 = \frac{\omega^2}{c^2}. \]

A solution of Eq. 2.7 propagating in z direction is:

\[ \mathcal{E}(r) = \mathcal{E}_0(r)e^{-ikz}, \quad (2.8) \]

When Eq. 2.8 is substituted into Eq. 2.7,

\[ \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \mathcal{E}_0(r)e^{-ikz} + k^2 \mathcal{E}_0(r)e^{-ikz} = 0. \quad (2.9) \]

Eq. 2.8 can be rearranged as:

\[ \left( \nabla_T + \frac{\partial^2}{\partial z^2} \right) \mathcal{E}_0(r) - 2ik \frac{\partial \mathcal{E}_0}{\partial z} = 0. \quad (2.10) \]

Here \( \nabla_T \) is:

\[ \nabla_T = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}. \]

When the change of the electric field changes slowly for the propagation direction and satisfy conditions such that:

\[ \lambda \left| \frac{\partial \mathcal{E}_0}{\partial z} \right| \ll |\mathcal{E}_0|, \quad \lambda \left| \frac{\partial^2 \mathcal{E}_0}{\partial z^2} \right| \ll \left| \frac{\partial \mathcal{E}_0}{\partial z} \right|. \]

Eq. 2.10 can be approximated in the form of the paraxial ray equation as:

\[ \left( \nabla_T - 2ik \frac{\partial}{\partial z} \right) \mathcal{E}_0(r) = 0. \quad (2.11) \]

Putting \( \mathcal{E}_0(r) \) as:

\[ \mathcal{E}_0(r) = Ae^{-ip(z)}e^{-i\frac{k(x^2+y^2)}{2q(z)}}, \quad (2.12) \]

Eq. 2.11 is written:

\[ \nabla_T \mathcal{E}_0 - 2ik \frac{\partial \mathcal{E}_0}{\partial z} = A \left[ \frac{k^2}{q^2}(x^2+y^2)\left( \frac{\partial q}{\partial z} - 1 \right) - 2k \left( \frac{\partial p}{\partial z} \right. \left. - i \frac{q}{p} \right) \right] e^{-i\frac{k(x^2+y^2)}{2q(z)}} e^{-ip(z)} \]

\[ = 0. \quad (2.13) \]

In order to satisfy Eq. 2.13,

\[ \frac{\partial q}{\partial z} = 1, \quad (2.14) \]

and

\[ \frac{\partial p}{\partial z} = -i \frac{q}{p}. \quad (2.15) \]
A solution of Eq. 2.14 and Eq. 2.15 are:

\[ q(z) = q_0 + z, \]
\[ q(0) = q_0, \]

and

\[ p(z) = -i \ln \frac{q_0 + z}{q_0}, \quad (2.16) \]
\[ p(0) = 0. \]

Because the exponential part of Eq. 2.12 is similar to Gaussian distribution, let transverse distribution of intensity of Eq. 2.12 be:

\[ I(r) = |\mathcal{E}(r)|^2 \propto e^{-2x^2+y^2/w_0^2} \]

\[ w \] is the spot size of the laser. In general, \( q \) is imaginary and \( q_0 \) should be pure imaginary number at \( z = 0 \) so that Eq. 2.12 may satisfy Eq. 2.17.

\[ \frac{1}{q_0} = -\frac{2i}{kw_0^2} = \frac{i \lambda}{\pi nw_0^2}, \quad (2.17) \]

where

\[ k = \frac{2\pi n}{\lambda} \]

and \( w(0) = w_0 \).

When we substitute Eq. 2.16 and Eq. 2.17 for Eq. 2.12 as follows;

\[ \mathcal{E}_0(r) = A \exp \left[ - \ln \left( 1 + \frac{z}{q_0} \right) \right] \exp \left( -i \frac{k(x^2 + y^2)}{2(q_0 + z)} \right) \]

\[ = A \exp \left[ - \ln \left( 1 + \frac{i\lambda z}{\pi nw_0^2} \right) \right] \exp \left( -i \frac{k(x^2 + y^2)}{2\left(\frac{\pi nw_0^2}{\lambda} + z\right)} \right) \]

\[ = A \frac{1}{\sqrt{1 + \frac{\lambda^2 z^2}{\pi^2 w_0^4 n^2}}} \exp \left\{ \left[ i \tan^{-1} \left( \frac{\lambda z}{\pi w_0^2 n} \right) \right] \right\} \]

\[ \times \exp \left\{ - \frac{x^2 + y^2}{w_0^2[1 + (\frac{\lambda z}{\pi w_0^2 n})^2]} - \frac{i k(x^2 + y^2)}{2z[1 + (\frac{\pi w_0^2 n}{\lambda z})^2]} \right\}. \]

\[ w^2(z), R(z), \eta(z) \text{ and } z_0 \text{ are calculated as; } \]

\[ w^2(z) = w_0 \left[ 1 + \left( \frac{\lambda z}{\pi w_0^2 n} \right)^2 \right] = w_0^2 \left( 1 + \frac{z^2}{z_0^2} \right), \]

\[ R(z) = z \left[ 1 + \left( \frac{\pi w_0^2 n}{\lambda z} \right)^2 \right] = z \left( 1 + \frac{z_0^2}{z^2} \right), \]

\[ \eta(z) = \tan^{-1} \left( \frac{\lambda z}{\pi w_0^2 n} \right) = \tan^{-1} \left( \frac{z}{z_0} \right), \quad (2.18) \]

\[ z_0 \equiv \frac{\pi w_0^2 n}{\lambda}. \]
2.2. PRINCIPLE OF LASER

Finally, the electromagnetic field with Gaussian shape transverse profile in the form of Eq. (2.6) can be written as;

$$E(x, y, z, t) = A \frac{w_0}{w(z)} e^{iwt} \exp \left[ -i(kz - \eta(z)) - i \frac{k(x^2 + y^2)}{2q(z)} \right]$$

$$= A \frac{w_0}{w(z)} e^{iwt} \exp \left[ -i(kz - \eta(z)) - (x^2 + y^2) \left( \frac{1}{w^2(z)} + \frac{ik}{2R(z)} \right) \right]$$

(2.19)

Eq. (2.19) shows the fundamental Gaussian beam. $w(z)$ and $R(z)$ are the spot size and the curvature radius of the equi-phase plane at $z$. $z_0$, named as Rayleigh range, is a region where the beam is focused around its focal point. $\eta(z)$, Gouy phase, is an additional phase shift to the propagation of the wave when it passes through the focal point [34][35].

Fig. 2.9 shows the spot size of Gaussian beam around the beam waist. The black line shows the spot size of Gaussian beam. The blue line shows the spot size calculated with geometrical optics as;

$$w_{geo}(z) = w_0 \frac{z}{z_0}.$$  (2.20)

The red line shows Rayleigh range. For $|z| > z_0$, the Gaussian beam has the constant divergence as shown in Eq. (2.20).

![Figure 2.9: Spot size of Gaussian beam and Rayleigh range.](image)

The underlined part of Eq. (2.19) is written as;

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i \frac{\lambda}{\pi nw^2(z)}.$$  (2.21)
Hereafter, \( q(z) \) is referred as Gaussian beam parameter.

In general, the solution for Gaussian beam is written in the form of Hermite-Gaussian mode as \( \text{[33]} \):

\[
E_{l,m}(x, y, z) = E_0 \frac{w_0}{w(z)} H_l \left( \sqrt{2} \frac{x}{w(z)} \right) H_m \left( \sqrt{2} \frac{y}{w(z)} \right) \\
\times \exp \left[ -ik \frac{x^2 + y^2}{2q(z)} - ikz + i(l + m + 1)\eta \right]
\]

\[
= E_0 \frac{w_0}{w(z)} H_l \left( \sqrt{2} \frac{x}{w(z)} \right) H_m \left( \sqrt{2} \frac{y}{w(z)} \right) \\
\times \exp \left[ -\frac{x^2 + y^2}{w^2(z)} - i \frac{x^2 + y^2}{2R(z)} - ikz + i(l + m + 1)\eta \right],
\]

(2.22)

where \( H_l \) and \( H_m \) is Hermite polynomial of \( l \)th and \( m \)th and the higher mode Gaussian beam with \( l \) and \( m \) is written as TEM\(_{lm}\) (Transverse Electro-Magnetic).

2.2.2.1 \( \text{ABCD law} \)

The propagation of Gaussian beam is described by transfer matrices. Specific form of the matrices for typical optical elements are summarized in Table 2.1.

When Gaussian beam with the beam parameter \( q_1 \) (see Eq. 2.21) pass through optical component of \( \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} \), the beam parameter, \( q_1 \), becomes \( q_2 \) as;

\[
q_2 = \frac{A_1 q_1 + B_1}{C_1 q_1 + D_1}.
\]

Since the transfer matrix is multiplicative, a Gaussian beam parameter \( q \) passing through multiple optical elements can be;

\[
q_3 = \frac{A_2 q_2 + B_2}{C_2 q_2 + D_2} = \frac{A_3 q_1 + B_3}{C_3 q_1 + D_3},
\]

where \( \begin{pmatrix} A_3 & B_3 \\ C_3 & D_3 \end{pmatrix} \) is;

\[
\begin{pmatrix} A_3 & B_3 \\ C_3 & D_3 \end{pmatrix} = \begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix} \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix}
\]

Thus, the beam parameters passing through optical elements are calculated with the \( \text{ABCD law} \) and is used for the design of the optical setup for the experiments.
2.3. PRINCIPLE OF OPTICAL RESONANT CAVITY

<table>
<thead>
<tr>
<th>Optical component</th>
<th>Image</th>
<th>Ray matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>The ray passes through distance of $d$ with homogeneous refractive index.</td>
<td><img src="image1.png" alt="Image" /></td>
<td>$(\begin{pmatrix} 1 &amp; d \ 0 &amp; 1 \end{pmatrix})$</td>
</tr>
<tr>
<td>The ray passes through thin lens of $f$ as focal length.</td>
<td><img src="image2.png" alt="Image" /></td>
<td>$(\begin{pmatrix} 1 &amp; 0 \ -\frac{1}{f} &amp; 1 \end{pmatrix})$</td>
</tr>
<tr>
<td>The ray enters from the medium of refractive index $n_1$ to the medium of refractive index with curvature radius $\rho$.</td>
<td><img src="image3.png" alt="Image" /></td>
<td>$(\begin{pmatrix} 1 &amp; 0 \ \frac{n_2-n_1}{n_2 \rho} &amp; \frac{n_1}{n_2} \end{pmatrix})$</td>
</tr>
<tr>
<td>The ray reflects with the concave mirror of curvature radius $\rho$.</td>
<td><img src="image4.png" alt="Image" /></td>
<td>$(\begin{pmatrix} 1 &amp; 0 \ -\frac{1}{\rho} &amp; 1 \end{pmatrix})$</td>
</tr>
</tbody>
</table>

Table 2.1: Ray matrix of each optical components

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</tr>
</tbody>
</table>

### 2.3 Principle of optical resonant cavity

#### 2.3.1 Principle of optical resonant cavity as wave

We consider the case that a plane wave is injected perpendicularly to the mirrors of a Fabry-Perot cavity which consists of two parallel mirrors. The amplitude transmission and reflection ratio of two mirrors, denoted as $t_1$ and $t_2$, are assumed to be $r_1$, $r_2$ respectively (see Fig. 2.10). When the amplitude of the injected plane wave is $E_{in}$, the amplitudes of the reflected wave $E_{rn}$ is expressed as:

$$E_{r1} = r_1 E_{in},$$

$$E_{rn(n>2)} = t_1^2 r_2 (r_1 r_2 e^{i\Delta \theta})^{n-2} E_{in},$$

and $E_{tn}$ is amplitudes of transmitted waves expressed as:

$$E_{tn} = t_1 t_2 (r_1 r_2 e^{i\Delta \theta})^{n-1} E_{in}.$$
Figure 2.10: Model of Fabry-Perot cavity. Electric field amplitude \( E_{\text{in}} \) is incident to two plate which are transmission \( t_1, t_2 \), reflection \( r_1, r_2 \).

Therefore, the total amplitudes of the reflected and the transmitted wave is described as;

\[
E_r = \sum_n E_{\text{rn}} = \left\{ r_1 - t_1^2 r_2 e^{i\Delta \theta} \left[ 1 + r_1 r_2 e^{i\Delta \theta} + (r_1 r_2 e^{i\Delta \theta})^2 + \cdots \right] \right\} E_{\text{in}}
= \left( r_1 - t_1^2 r_2 e^{i\Delta \theta} \frac{1}{1 - r_1 r_2 e^{i\Delta \theta}} \right) E_{\text{in}}, \tag{2.23}
\]

\[
E_t = \sum_n E_{\text{tn}} = t_1 t_2 \left[ 1 + r_1 r_2 e^{i\Delta \theta} + (r_1 r_2 e^{i\Delta \theta})^2 + \cdots \right] E_{\text{in}}
= \frac{t_1 t_2}{1 - r_1 r_2 e^{i\Delta \theta}} E_{\text{in}},
\]

where \( \Delta \theta \) is the phase difference of the electromagnetic wave for one round-trip between two mirrors.

The sum of electric field parallel to the transmitted wave and one parallel to the reflected wave (anti-parallel to the transmitted wave) in the optical cavity. \( E_{\text{s--outward}} \) and \( E_{\text{s--return}} \) can be written as;

\[
E_{\text{s--outward}} = \frac{1}{t_2} E_t,
E_{\text{s--return}} = \frac{1}{t_1} E_{\text{rn(n>1)}}.
\]

Thus, the amplitude of the light wave in the cavity is;

\[
E_s = \sum_n E_{\text{sn}} = E_{\text{s--outward}} + E_{\text{s--return}}
= \frac{t_1}{1 - r_1 r_2 e^{i\Delta \theta}} E_{\text{in}} + \frac{t_1 r_2}{1 - r_1 r_2 e^{i\Delta \theta}} E_{\text{in}}
= \frac{1 + r_2}{1 - r_1 r_2 e^{i\Delta \theta}} t_1 E_{\text{in}} = (1 + r_2) \frac{E_t}{t_2}.
\]
Ratios of intensities of light waves, reflected in, transmitted from and stored in the cavity are:

\[
\frac{I_r}{I_{in}} = \frac{E_r E_r^*}{E_{in} E_{in}^*} = R_1 + T_1 - \frac{T_1 (1 - R_1 R_2 - T_1 R_2)}{(1 - \sqrt{R_1 R_2})^2 + 4 \sqrt{R_1 R_2} \sin^2(\frac{\Delta \theta}{2})},
\]

\[
\frac{I_t}{I_{in}} = \frac{E_t E_t^*}{E_{in} E_{in}^*} = T_1 T_2 (1 - \sqrt{R_1 R_2})^2 + 4 \sqrt{R_1 R_2} \sin^2(\frac{\Delta \theta}{2}),
\]

\[
\frac{I_s}{I_{in}} = \frac{E_s E_s^*}{E_{in} E_{in}^*} = \frac{1 + R_2}{T_2} \frac{I_t}{I_{in}},
\]

where \(T_n\) and \(R_n\) are the transmission and reflection ratio for intensities i.e., \(T_n = |t_n|^2\), \(R_n = |r_n|^2\).

When the cavity resonates with the incident wave, \(\Delta \theta = 0\), the relation between the angular frequency of the incident wave and the interval of mirrors is:

\[\omega_m = m \frac{\pi c}{L}.\]

The separation of the resonant angular frequency between resonance is:

\[\Delta \omega = \omega_{m+1} - \omega_m = \frac{\pi c}{L}.
\]

\(\Delta \omega\) is called the FSR (free spectral range).

\(\Delta \theta\) dependence of intensity of the transmitted light of a Fabry-Perot cavity is shown in Fig. 2.11. Intensities of the transmitted and the stored light are peaked at the resonance. On the other hand, the intensity of the reflected light is minimum at the resonance.

Figure 2.11: Phase dependence of intensity of the transmitted light of the optical resonant cavity.

The intensity of the transmitted light as a function of the phase or the angular frequency is called the Airy function. The FWHM of the peak
The sharpness of the resonance can be expressed by the finesse $F$ defined as:

$$F \equiv \frac{\Delta \omega}{\delta \omega} = \frac{\pi \sqrt{R}}{1 - R}, \quad (2.24)$$

The injected electromagnetic wave is accumulated in the resonant cavity and the enhancement factor for the intensity is $F/\pi$.

### 2.3.2 Principle of optical resonant cavity with Gaussian beam

For a 2 mirror optical resonant cavity, the Gaussian beam parameter should be the same after a round trip to keep resonant condition. For a cavity of which two mirrors separates $L_{\text{cav}}$, the ray matrix starting from a mirror is written as follows:

$$
\begin{pmatrix}
A & B \\
C & D
\end{pmatrix}
= \begin{pmatrix}
1 & 0 \\
-\frac{2}{\rho_l} & 1
\end{pmatrix}
\begin{pmatrix}
1 & L_{\text{cav}} \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
-\frac{2}{\rho_r} & 1
\end{pmatrix}
\begin{pmatrix}
1 & L_{\text{cav}} \\
0 & 1
\end{pmatrix}
= \begin{pmatrix}
1 - \frac{2L_{\text{cav}}}{\rho_l \rho_r} & \frac{2L_{\text{cav}}^2(\rho_r - L_{\text{cav}})}{\rho_l \rho_r} \\
\frac{-2(-2L_{\text{cav}} + \rho_l + \rho_r)}{\rho_l \rho_r} & 4L_{\text{cav}}^2 + \rho_l \rho_r - 2L_{\text{cav}}(\rho_l + 2\rho_r)
\end{pmatrix}, \quad (2.25)
$$

where $\rho_l$ and $\rho_r$ stand for curvature of two mirrors.

When the cavity is on resonance, Gaussian beam parameter $q$ is:

$$q = \frac{Aq + B}{Cq + D}.$$

Thus $q$ must satisfy:

$$\frac{1}{q} = \frac{D - A}{2B} \pm \frac{\sqrt{(D - A)^2 + 4BC}}{2B}.$$

Because Eq. 2.25 satisfies $AD - BC = 1$, $1/q$ is:

$$\frac{1}{q} = \frac{D - A}{2B} \pm \frac{\sqrt{(D + A)^2 - 4}}{2B} = \frac{1}{R} \pm i \frac{\lambda}{\pi nw^2}.$$

Then:

$$R = \frac{2B}{D - A},$$

$$w^2 = \frac{\lambda}{\pi} \frac{2|B|}{\sqrt{4 - (D + A)^2}}. \quad (2.26)$$
2.3. PRINCIPLE OF OPTICAL RESONANT CAVITY

In order for to be a real,

\[ 4 - (D + A)^2 > 0 \]
\[ -2 < D + A < 2 \]

The condition for A and D can be converted to those for L and ρs with Eq. 2.25 as;

\[ 0 < \left( \frac{L_{\text{cav}}}{\rho_l} - 1 \right) \left( \frac{L_{\text{cav}}}{\rho_r} - 1 \right) < 1. \]

Fig. 2.12 shows area of Eq. 2.25.

Figure 2.12: The optical resonant cavity has resonance solution, provided that blue area.

2.3.3 Higher Gaussian beam mode in the optical resonant cavity

When the laser light is put into an optical resonant cavity with an angle with respect to the optical axis of the cavity, higher transverse modes, which are cases for \( l, m \neq 0 \) in Eq. 2.22, are excited. For the purpose of laser Compton interaction, the cavity is usually tuned to maximize the amplitude of the fundamental \( (l = m = 0) \) mode. However, 1st mode can be used to monitor the resonant condition of the optical resonant cavity which is described in detail in § 2.2.2. Transverse profiles of Gaussian beam mode are shown Fig. 2.13, Fig. 2.14, Fig. 2.15, Fig. 2.16.

From Eq. 2.22, the electric field of a higher mode near the laser axes is [33, 36];

\[ \mathcal{E}_{l,m}(z) \propto \exp[-ikz + i(l + m + 1)z]. \] (2.27)
From Eq. (2.18), \( \eta = \tan^{-1}\left(\frac{z}{z_0}\right) \), the phase factor in Eq. (2.27), \( \Theta_{lm}(z) \) is;

\[
\Theta_{lm}(z) = k z - (l + m + 1) \tan^{-1}\left(\frac{z}{z_0}\right). \tag{2.28}
\]

When the optical resonant cavity resonates with the incident lasers, phase difference at two mirrors becomes;

\[
\Theta_{lm}(z_2) - \Theta_{lm}(z_1) = n \pi \quad (n : \text{integer}). \tag{2.29}
\]

\( z_1 \) and \( z_2 \) are position of z axis at mirrors of the optical resonant cavity. From Eq. (2.28) and Eq. (2.29)

\[
k_n L_{cav} - (l + m + 1) \left[ \tan^{-1}\left(\frac{z_2}{z_0}\right) - \tan^{-1}\left(\frac{z_1}{z_0}\right) \right] = n \pi,
\]

where \( L_{cav} \) the distance between two mirrors, \( z_2 - z_1 \). The separation of the resonant wave number is;

\[
k_{n+1} - k_n = \frac{\pi}{L_{cav}}
\]
2.3. PRINCIPLE OF OPTICAL RESONANT CAVITY

because

\[ k_{n+1} L_{cav} - (l + m + 1) \left[ \tan^{-1} \left( \frac{z_2}{z_0} \right) - \tan^{-1} \left( \frac{z_1}{z_0} \right) \right] = (n + 1) \pi, \]

and difference of angular frequency is;

\[ \Delta \omega_n = \omega_{n+1} - \omega_n = \frac{\pi c}{L_{cav}}. \] (2.30)

For different \( l, m \) but with the same \( n \), the difference of wave number is;

\[ (k_1 - k_2) L_{cav} = [(l+m+1)_1 - (l+m+1)_2] \left[ \tan^{-1} \left( \frac{z_2}{z_0} \right) - \tan^{-1} \left( \frac{z_1}{z_0} \right) \right], \]

in the form of the angular frequency;

\[ \Delta \omega_{lm} = \omega_{l+m+1} - \omega_{l+m} = \frac{c}{L_{cav}} \left[ \tan^{-1} \left( \frac{z_2}{z_0} \right) - \tan^{-1} \left( \frac{z_1}{z_0} \right) \right]. \] (2.31)

The terms in the bracket is;

\[ \tan^{-1} \left( \frac{z_2}{z_0} \right) - \tan^{-1} \left( \frac{z_1}{z_0} \right) = \cos^{-1} \sqrt{\left( 1 - \frac{L_{cav}}{\rho_2} \right) \left( 1 - \frac{L_{cav}}{\rho_1} \right)} \]

From Eq. (2.30) and Eq. (2.31);

\[ \omega_{lmn} = \frac{c}{L_{cav}} \left[ n \pi + (l + m + 1) \cos^{-1} \sqrt{\left( 1 - \frac{L_{cav}}{\rho_2} \right) \left( 1 - \frac{L_{cav}}{\rho_1} \right)} \right]. \] (2.32)

Fig. 2.17 shows excited transverse modes written in by Eq. (2.32). Since \( TEM_{lm} \) and \( TEM_{ml} \) mode degenerates, it is called \((l + m)_{th}\) mode.

When the curvature of the two mirrors are the same, i.e, \( \rho = \rho_1 = \rho_2 \), the phase difference between \((l + m)_{th}\) mode and \((l + m + 1)_{th}\) mode is;

\[ \Delta \Theta = \Theta_{(l+m+1)h} - \Theta_{(l+m)h} = \cos^{-1} \left( 1 - \frac{L_{cav}}{\rho} \right). \]

From Eq. (2.26) and Eq. (2.25), the waist size is;

\[ w_0^2 = \frac{\lambda L_{cav}}{2\pi} \sqrt{\frac{1 - \cos \Delta \Theta}{1 + \cos \Delta \Theta}}. \] (2.33)

We can observe excited transverse mode, as shown in Fig. 2.17, with the transmitted and reflected light when the cavity length or the laser frequency is scanned. With Eq. (2.33), we can obtain waist size in the optical resonant cavity by phase difference between higher modes of the transmitted light or the reflected light.
2.4 Pulse stacking to the optical resonant cavity

Because the electron beam from accelerators for laser Compton scattering has bunch structure, it is more efficient to use the pulsed laser for increasing the laser intensity that contributes to the collision with the electron.

In Fig. 2.18-(a), red lines indicate resonance condition of the optical resonant cavity. Blue line in Fig. 2.18-(b) is the angular frequency distribution of a laser pulse. In general, the angular frequency of pulsed laser is continuous spectrum, while the resonance condition of the cavity is discrete distribution. Therefore, the pulsed laser cannot be resonated with the optical cavity.

2.4.1 Mode locked pulsed laser

The solution of the pulse stacking is to use a mode locked pulsed laser. As mentioned in §2.2.1 the laser oscillator has the optical cavity in side. Hereafter, the optical cavity in the laser oscillator is referred as the laser oscillator cavity. There are many angular frequency modes in the oscillator cavity, provided that a gain bandwidth of the laser medium is wider than the separation of the resonance condition. The electric field, $E(t)$, in the laser oscillator cavity is written as;

$$E(t) = \sum_{m} E_m e^{i[(\omega_0+m\Delta\omega)t+\phi_m]}.$$  \hspace{1cm} (2.34)
2.4. PULSE STACKING TO THE OPTICAL RESONANT CAVITY

Figure 2.18: The resonance condition of the optical resonant cavity and the angular frequency distribution of a laser pulse. Red lines of (a) indicate resonance condition of the optical resonant cavity. Blue line of (b) is the angular frequency distribution of a laser pulse.

Separation of angular frequency $\Delta \omega$ is defined to be:

$$\Delta \omega \equiv \omega_{n+1} - \omega_n = \frac{\pi c}{L_{\text{laser}}}$$

here $L_{\text{laser}}$ is the cavity length in the oscillator. For simplicity $E_m, \phi_m$ are assumed to be a constant and independent of $m$. If $N$ modes are excited in the oscillator cavity, Eq. 2.34 is written as;

$$E(t) = NE^{i(\omega_0 t + \phi)} \sum_{m=-\frac{N-1}{2}}^{\frac{N-1}{2}} e^{im\Delta \omega t}$$

$$= NE^{i(\omega_0 t + \phi)} e^{i\frac{1}{2}N\Delta \omega t} \left(e^{iN\Delta \omega t} - 1\right) e^{i\Delta \omega t} - 1$$

$$= NE^{i(\omega_0 t + \phi)} \sin\left(\frac{N\Delta \omega t}{2}\right) \sin\left(\frac{\Delta \omega t}{2}\right).$$

The intensity of laser beam $I(t)$ is;

$$I(t) = E(t)E^*(t) \propto \frac{\sin^2\left(\frac{N\Delta \omega t}{2}\right)}{\sin^2\left(\frac{\Delta \omega t}{2}\right)}.$$

The the temporal profile of the laser $I(t)$ is a pulsed shape as shown in Fig. 2.19. The peak intensity the pulse is proportional to $N^2$. 
2.4.2 Pulse stacking of mode locked laser

In general, the optical resonant cavities must satisfy a condition of its length $L_{cav}$ as;

$$L_{cav} = n \frac{\lambda}{2} \quad (n: \text{natural number}),$$

to accumulate the laser wave of wave length $\lambda$. An additional condition between the pulse stacking cavity and the cavity in the laser oscillator is required

$$L_{cav} = mL_{laser} \quad (m: \text{natural number}), \quad (2.35)$$

to ensure all frequency component from the laser are resonated in the optical resonant cavity. The condition of Eq. (2.35) is illustrated in Fig. 2.20. In Fig. 2.20-(a), the red lines indicate resonance condition of the optical resonant cavity. The blue lines in Fig. 2.20-(b) are the angular frequency distribution of a mode locked laser. If Eq. (2.35) satisfies, mode locked laser pulses can be resonant in the optical resonant cavity.

2.5 The feedback control system

In this experiment, the optical resonant cavity is required to satisfy resonance condition. In addition, laser pulse timing is required to synchronize with the electron beam. The required accuracy for cavity length is sub nanometer and for timing is picosecond. In order to achieve these conditions simultaneously a feedback system is constructed.
2.5. THE FEEDBACK CONTROL SYSTEM

The word feedback means that the output signal of a system is returned and fed into the input as schematically shown in Fig. 2.21. The output, which has information of the system performance, is fed back to the input, which is a reference signal at which the system should perform, and the difference of the input and the output is put into the filter. To stabilize the system according to the input signal, the deviation between input and output signal was taken and put into the filter. The purpose of the filter is to process the difference of the input and the output, called the error signal, and makes a control signal appropriate to obtain desired system performance.

In this experiment, we used a PID control as the filter. The PID is abbreviation of Proportional, Integral and Differential. An output signal $V_{\text{out}}(t)$ of the PID control for an input $V_{\text{in}}(t)$ is expressed as;

$$V_{\text{out}}(t) = K_P V_{\text{in}}(t) + K_I \int_0^t V_{\text{in}}(T) dT + K_D \frac{dV_{\text{in}}(t)}{dt}, \quad (2.36)$$
where $K_P$, $K_I$ and $K_D$ are gain of Proportional, Integral and Differential components of the PID respectively. Here we adopt a time constant $\tau_I = \frac{K_P}{K_I}$ and $\tau_D = \frac{K_D}{K_P}$. Eq. 2.36 is written as:

$$V_{\text{out}}(t) = K_P \left( V_{\text{in}}(t) + \frac{1}{\tau_I} \int_0^t V_{\text{in}}(T) dT + \tau_D \frac{dV_{\text{in}}(t)}{dt} \right).$$

In general, integrations and differentiations are implemented by system with time constants, $\tau$.

![Figure 2.22: A simple example of the PID feedback loop.](image)

From Eq. 2.36, the transfer function of the PID system can be defined using the prescription of the Laplace transformation as;

$$\frac{v_{\text{out}}(s)}{v_{\text{in}}(s)} = K_P \left( 1 + \frac{s}{\tau_I} + \tau_D s \right),$$

where $v$ is function of Laplace transformed $V$ as;

$$v(s) = \int_0^\infty V(t)e^{-st}dt.$$

The transfer function of the PID circuit with the expression of the integration and differentiation of the time constant $\tau_I$ and $\tau_D$ is;

$$\frac{v_{\text{out}}(s)}{v_{\text{in}}(s)} = K \left( K_P + \frac{s}{\tau_I} + \frac{\tau_D s}{1 + \tau_D s} \right)$$

We can adjust gains of the integration and differentiation independently of $K_P$.

2.5.2 Feedback control technique for resonance the optical resonant cavity

We control cavity length with an error signal which has the phase information $\Delta \theta$ (see Eq. 2.23) by the reflected electromagnetic field from the optical resonant cavity. Since the phase changes the sign across the resonance condition, it can be used to monitor the status of the cavity including the direction to the resonance.
2.5. THE FEEDBACK CONTROL SYSTEM

2.5.2.1 Tilt locking technique for resonance the optical resonant cavity

In order to measure the phase information described in above, the tilt lock technique was adopted in this experiment [37]. In tilt locking technique, the phase inversion on the resonance of the TEM_{0th} mode is monitored by the interference between TEM_{0th} and TEM_{1st} mode.

Transverse distribution of the electric magnetic field and the laser intensity around resonance of the optical resonant cavity is shown in Fig. 2.23.

<table>
<thead>
<tr>
<th>off resonance ((L &lt; L_{on}))</th>
<th>on resonance ((L = L_{on}))</th>
<th>off resonance ((L &gt; L_{on}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electromagnetic field [a.u.]</td>
<td>Electromagnetic field [a.u.]</td>
<td>Electromagnetic field [a.u.]</td>
</tr>
<tr>
<td>Position of transverse axis [a.u.]</td>
<td>Position of transverse axis [a.u.]</td>
<td>Position of transverse axis [a.u.]</td>
</tr>
<tr>
<td>Laser intensity [a.u.]</td>
<td>Laser intensity [a.u.]</td>
<td>Laser intensity [a.u.]</td>
</tr>
<tr>
<td>Position of transverse axis [a.u.]</td>
<td>Position of transverse axis [a.u.]</td>
<td>Position of transverse axis [a.u.]</td>
</tr>
</tbody>
</table>

Figure 2.23: Electric magnetic field and laser intensity around resonance of the optical resonant cavity.

In Fig. 2.23, figures in the center show that the length of the optical resonant cavity satisfies the resonance condition, \(L_{on}\). Left figures show the case for the length of the optical resonant cavity is smaller than \(L_{on}\), while right ones show the length of the optical resonant cavity is longer than \(L_{on}\). For “electromagnetic field”, red lines show the electromagnetic field distribution of TEM_{0th}. Blue lines show the electromagnetic field distribution of TEM_{1st}. Horizontal axis of figures are the transverse axis centered at the optical axis of the cavity. Intensities are square of sum of the electric magnetic fields.

From Fig. 2.24, it is seen that difference between right side and left side of laser intensity distribution has information for the resonance condition of the cavity as [37].
\[ I = \left| \int_{-\infty}^{0} E_0^*(x)E_1(x)\,dx \right| - \left| \int_{0}^{\infty} E_0^*(x)E_1(x)\,dx \right|, \quad (2.37) \]

which can be measured by a position sensitive photo detector such as a split photo diode. Fig. 2.25 shows a schematic of the optics for tilt locking technique. As seen from the figure, \( I \) in Eq. 2.37 is a differential signal for the length of the optical cavity and is used for locking the length of the cavity at the resonance condition.

Figure 2.24: The difference between right and left by the split photo diode.

Figure 2.25: A scheme of the optics for tilt locking technique.

### 2.5.3 Timing control technique

The timing of the laser pulses must be synchronized with the electron bunches. We used the phase locked loop (PLL) technique for locking the laser pulse timing. The PLL is a technique for synchronizing to oscillators by measuring relative phase of oscillators. Fig. 2.26 shows basic scheme of the PLL. VCO is a voltage controlled oscillator in which output signal with a frequency \( \omega \) of;
\[ \omega = \omega_0 + V \delta \omega. \]

The reference signal \( f_{\text{ref}} \) and a VCO output signal \( f \) are written as:

\[
\begin{align*}
   f_{\text{ref}} & \propto \sin (\omega_{\text{ref}} t + \phi_{\text{ref}}), \\
   f & \propto \sin (\omega t + \phi).
\end{align*}
\]

\( f_{\text{ref}} \) and \( f \) are mixed at the phase detector as:

\[
   f_{\text{ref}} \times f \propto \cos (\omega_{\text{ref}} t + \phi_{\text{ref}} - \omega t - \phi) - \cos (\omega_{\text{ref}} t + \phi_{\text{ref}} + \omega t + \phi)
\]

The lower frequency was chosen by the low pass filter, and adjust the gain of loop which satisfies \( V \delta \omega = \omega_{\text{ref}} - \omega_0 \). Then the frequency of VCO becomes \( \omega_{\text{ref}} \). The phase difference becomes constant as frequency synchronized.
Chapter 3

Experimental Setup of Gamma Ray Generation with the Optical Resonant Cavity

In this experiment, gamma rays are generated by collision of electron beams and laser pulses accumulated in an optical resonant cavity. The optical resonant cavity was installed in the KEK-ATF damping ring (ATF-DR). A detector system was set up for a gamma ray detection. The setup was required to fulfill conditions such that:

- It should not interfere the ATF operation, particularly not affect the electron beam emittance.
- The optical resonant cavity need to be kept on resonance with the finesse expected by its design.
- The laser pulses have to be synchronized with electron bunches in the ATF-DR.
- The detector system has capability to measure the gamma ray yield and energy of laser Compton scattering.

In this chapter, we describe experimental setup to meet above requirements.

3.1 The KEK-ATF

The experiment was performed at the Accelerator Test Facility (ATF) in High Energy Accelerator Research Organization (KEK) in Japan. The ATF is a accelerator facility for a research and development of an ultra low emittance electron beam required for the International Linear Collider. The study at the ATF includes generation of the ultra low emittance beam as well as beam diagnostics and control techniques necessary
for the ILC beam. Fig. 3.1 shows the ATF layout and place of laser Compton interaction point and the gamma ray detector.

At the ATF, electrons are generated by 1.6 cell RF gun. They are accelerated to 1.28 GeV in the electron linear accelerator with nine 3 m long RF cavity of 2856 MHz. Two RF cavities of frequencies of 2851.6728 and 2860.3272 (2856±4.3272) MHz are used for energy correction for multi bunch operations. After accelerated to 1.28 GeV, the electron beam is injected to the Damping Ring (DR). RF frequency of the ATF-DR is 714 MHz. The revolution of the ATF-DR is 2.16 MHz due to its circumference of 138 m. After about 500,000 turns in the ATF-DR, the emittance of the beam is damped to desired level and is extracted to the extraction line for experiments with extracted beam. This operation is called the normal mode. In the normal mode, the electron beam is injected into the ATF-DR with repetition of several Hz. For laser Compton experiment, the ATF is operated as the storage mode. At the storage mode, electron beam is not extracted to extraction line and is kept circulating in the ATF-DR. After the circulation, the electron beam is dumped and new one is injected to the ring. The typical beam size is 100 µm (horizontal) × 10 µm (vertical) in the root mean square at the Compton interaction point. The electron beam consists of electron bunches called a train. The length of a bunch is 25 ps (7.5 mm) in the
root mean square, and bunch by bunch separation is 2.8 ns (0.84 m). The number of bunches in a train can be varied from 1 to 20 depending on the experiment. The maximum number of trains in the ATF-DR is 3 and the interval between train is 154 ns.

The ATF-DR requires very high vacuum of $10^{-7}$ Pa for generation low emittance beam. Thus, vacuum level of $10^{-6}$ is required for inside of the optical cavity. Table 3.1 summarizes parameters of the electron beam.

<table>
<thead>
<tr>
<th>electron beam at interaction point</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>beam energy</td>
<td>1.28 GeV</td>
</tr>
<tr>
<td>bunch spacing</td>
<td>2.8 ns</td>
</tr>
<tr>
<td>beam size</td>
<td>$100 , \mu m \times 10 , \mu m$</td>
</tr>
<tr>
<td>bunch length</td>
<td>25 ps</td>
</tr>
<tr>
<td>DR revolution</td>
<td>2.16 MHz</td>
</tr>
</tbody>
</table>

3.1.1 Beam monitor

Current monitors are used to measure the intensity of the electron beam in the ATF-DR. In this experiment, we use the DCCT and the WCM as current monitor. A streak camera is used to measure the length of a bunch.

3.1.1.1 The DCCT

The DCCT is abbreviation of a DC Current Transfer. Fig. 3.2 shows a schematic of the DCCT [38]. The electron beam passes the magnetic toroidal core which is excited by the sinusoidal current. The magnetic field in the magnetic toroidal core shifts up or down when the beam current passes through the magnetic toroidal core. The shift is detected with the coil, and the absolute value of the beam current is calculated. The time resolution of the DCCT is millisecond order.

3.1.1.2 The WCM

The WCM is abbreviation of Wall Current Monitor. Fig. 3.3 shows the schematic of the WCM. The beam pipe is delimited with an insulator and the two beam pipe is electrically connected by a resistor. The voltage is induced across the resistor when the electron beam passes through WCM. The time response is determined by the impedance of the insulator and a capacitance of the two beam pipes [39], and can be adjusted about 1 ns. Because the pulse height depends on the length of a electron bunch, it
CHAPTER 3. EXPERIMENTAL SETUP OF GAMMA RAY GENERATION WITH THE OPTICAL RESONANT CAVITY

is difficult to calculate the absolute value of the current. Fig. 3.4 shows the waveform of WCM. Each peak show the rate of each bunch current.

3.1.1.3 Streak camera

A streak camera is a device that monitors the time structure of light. Fig. 3.5 shows the schematic of a streak camera.

The incident light is converted to electrons at the photocathode, then electrons are swept by the sweeping electrode so that the time structure of electrons is converted to the transverse position of the electrons on a position sensitive device, (a Micro Channel Plate). In the ATF-DR, time duration of synchrotron radiation from an electron bunch is measured as the length of the electron bunch. Fig. 3.6 shows the image of streak camera. Vertical size of image shows the bunch length. Horizontal axis shows the revolution of the ATF-DR.
3.2 Optical setup

3.2.1 Laser oscillator

The laser oscillator used in this experiment is COUGAR manufactured by Time-Bandwidth Products, Inc. It uses Nd:YVO₄ as the lasing material and the wavelength is 1064 nm. The laser generates pulses of 28 nJ with a 357 MHz repetition. The repetition rate is chosen to be the same as the temporal spacing of electron bunches in the ATF ring. The pulse width is 5 ps in the root mean square. Laser beam position stability is 25 µrad/°C and output power stability is 0.1 % in RMS. Table 3.2 summarizes parameters of the laser oscillator.

<table>
<thead>
<tr>
<th>laser oscillator</th>
<th>mode lock</th>
<th>Nd:YVO₄</th>
<th>357 MHz</th>
<th>1064 nm</th>
<th>5 ps</th>
<th>420 mm</th>
<th>10 W</th>
</tr>
</thead>
</table>
3.2.2 Optical resonant cavity

The optical resonant cavity was installed at the straight section of the ATF-DR as shown in Fig. 3.1. It is a 2 mirrors Fabry-Perot type cavity and was placed in the vacuum chamber to keep vacuum level of the ATF beam line as described later in detail.

Fig. 3.7 shows the laser electron crossing angle dependence of the number of gamma rays calculated by a simulation code CAIN [43, 44], where the crossing angle of 0 degree means head on collision. The crossing angle should be small to increase the number of gamma rays. However, it is limited by the physical size of components such as mirrors, piezo actuators, an electron beam pipe, when the cavity is installed in the ATF-DR.

![Figure 3.7](image)

Figure 3.7: The crossing angle dependence of the number of gamma rays simulated [43] by simulation code CAIN [44]. The number of gamma rays is normalized by crossing angle = 0 degree.

Fig. 3.8 shows the arrangement of components around the mirror [45]. Yellow squares show piezo actuators. Dark blue shows mirrors which located inside piezo actuators. The beam pipe passes through the end cap of the optical resonant cavity as shown Fig. 3.9. The crossing angle of laser pulses and the electron beam is 12 degree which is determined by size of mirrors. A slit on the electron beam pipe is necessary when getting laser pulses through the electron beam. The slit should be small because the asperity of the beam pipe affects the electron beam. However, it is required to be large enough and it does not to affect the quality of the laser in the cavity. A calculation shows that the slit size should be more than 10σ of laser spot size [46]. Thus, the slit size is determined to be 35 mm (horizontal) × 5 mm (vertical) which meets requirement for laser.
3.2. OPTICAL SETUP

pulses and the electron beam.

Figure 3.8: The mirror holder. Yellow squares show piezo actuators. Dark blue shows mirrors. The mirrors are located inside of piezo actuators.

Figure 3.9: Schematic of the optical resonant cavity in the vacuum chamber. The orange line shows electron beam line. The red line shows the laser axis. The blue pieces show the spherical mirror holder. The green pieces show the piezo actuator holder. The beam pipe passes through the end cap of the optical resonant cavity.

The repetition rate of laser pulses is 357 MHz and the length of the laser oscillator cavity is 420 mm. The length of the optical resonant cavity is chosen to be 420 mm by Eq. 2.35. The curvature radius of concave spherical mirrors is 210.5 mm. The reflectance of intensity of two mirrors is 99.64 % and the transmission of intensity of two mirrors is 0.336 % [47]. By Eq. 2.24, the enhancement factor of the optical resonant cavity is calculated to be:

$$\frac{\text{Finesse}}{\pi} = \frac{780}{\pi} \sim 250$$

The waist size is expected to be $2\sigma = 60 \mu m$ in the optical resonant cavity by the calculation. A 15 mm long piezo actuator is installed for feedback control at the mirror holder of injection side. Here after, it is referred as the short piezo. The displacement of the short piezo actuator is about 4 $\mu m$ with 1 kV. 85 mm long piezo, referred as the long piezo, actuator
is installed for a coarse adjustment at the mirror holder of transmission side. The displacement of the long piezo is about 20 $\mu$m with 1 kV.

3.2.3 Laser optics

The Gaussian beam parameter (Eq. 2.21) must be matched with the parameter of resonant condition of the optical resonant cavity to stack laser pulses (Eq. 2.26). Thus we tuned the Gaussian beam parameter by the telescope which was constructed by convex lenses and concave lenses. Arrangement of optical components of the telescope on the optical table is shown in Fig. 3.10 and Fig. 3.11.

Figure 3.10: Laser optics. These were arranged to match the optical resonant cavity in limited table space.

3.2.3.1 Mode matching

The alignment of mirrors mounted on the cavity is not possible from outside of the vacuum chamber after the evacuation of the chamber. Therefore, the mode matching of the optical resonant cavity including alignment of the mirror on the cavity was performed in the atmosphere before the evacuation of the chamber. First, we aligned the laser axis to the cylinder axis of the optical resonant cavity without mounting cavity mirrors. Next, we attached the end cap, the mirror and its holder of the cavity, of the laser injection side. The optical components were aligned so as to make the path of the injection laser and the reflection from
the injection mirror in a line. The divergence of the laser profile of the injection and the reflection from the injection mirror were tuned to be the same by the telescope. By these procedure, it was assured that the equi-phase plane of the laser beam was matched with the curvature of the injection mirror.

Next, we mounted the end cap of the transmission side of the cavity. The transverse position of the end cap was fixed at where the intensity of transmission light become maximum when the length of the cavity was scanned by the short piezo.

Finally, the laser axis and the laser divergence was fine tuned to maximize fundamental Gaussian mode of the transmitted light.

3.2.3.2 Laser stacking condition

Fig. 3.12 shows the intensity of the transmitted light of the optical resonant cavity when scanning the length of the cavity. In this condition, we measured the FSR and the FWHM when the length of the cavity is scanned by the short piezo. The finesse was about 1000 (finesse = FSR/FWHM) by Eq. 2.24. Measured waist size was about 60 µm (2σ) by the phase difference of transmitted light (Eq. 2.33). The laser intensity in the cavity at the peak was about 500 W which was estimated by the transmitted power and the transmission rate of the mirror. Table 3.3 summarizes parameters of the optical resonant cavity.

3.2.4 Vacuum chamber

The optical resonant cavity was placed in the vacuum chamber to keep the vacuum level in the electron beam pipe at tolerated level. Fig. 3.13
CHAPTER 3. EXPERIMENTAL SETUP OF GAMMA RAY GENERATION WITH THE OPTICAL RESONANT CAVITY

Figure 3.12: The intensity of the transmitted light with scanning the length of the optical resonant cavity. (b) is a closeup of (a).

Table 3.3: Parameters of the optical resonant cavity.

<table>
<thead>
<tr>
<th>optical resonant cavity</th>
<th>type</th>
<th>2mirror Fabry-Perot</th>
</tr>
</thead>
<tbody>
<tr>
<td>cavity mirror</td>
<td>R=99.64 %, T=0.336 %</td>
<td></td>
</tr>
<tr>
<td>finesse</td>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>enhancement</td>
<td>250</td>
<td></td>
</tr>
<tr>
<td>waist size</td>
<td>60 µm (2σ)</td>
<td></td>
</tr>
<tr>
<td>cavity length</td>
<td>420 mm</td>
<td></td>
</tr>
<tr>
<td>stacking intensity</td>
<td>500 W</td>
<td></td>
</tr>
<tr>
<td>timing jitter of laser pulse</td>
<td>5 ps</td>
<td></td>
</tr>
</tbody>
</table>

is the picture of the optical resonant cavity in the vacuum chamber.

The vacuum level of $10^{-6}$ Pa in the vacuum chamber is required by the ATF operation. A baking technique is not applicable in this set up to improve the vacuum, because the high reflection mirrors and the piezo actuators are installed in the vacuum chamber. Especially, the piezo actuator could be damaged at temperature about 100 °C. Thus, we used two turbo molecular pumps and scroll pump before a starting the ATF operation. The Evacuation speed of turbo molecular pumps are 200 $l$/min and 300 $l$/min. The vacuum chamber was always vacuated by two 100 $l$/min ion pumps. The pressure in the Compton chamber was kept $10^{-6}$ Pa as shown in Fig. 3.14. We also confirmed that the chamber did not affect the average pressure of the ATF-DR of $10^{-7}$ Pa.
Figure 3.13: The optical resonant cavity in the vacuum chamber.

Figure 3.14: Vacuum level in the vacuum chamber and the ATF-DR average. The yellow line shows the vacuum level in the vacuum chamber. The red line shows the vacuum level of the ATF-DR average. The pressure in the Compton chamber satisfies the level that does not influence for the ATF operation because average pressure of the ATF-DR was kept $10^{-7}$ Pa.

The vacuum chamber was placed on the optical table as shown in Fig. 3.15. Fig. 3.15 shows the vacuum chamber (enclosed by the box) in the electron beam line. All components on the optical table were placed on the movable table. The position of the optical cavity (and the path of the laser) can be moved with the accuracy of 0.8 $\mu$m and 4.0 $\mu$m in vertical and horizontal direction respectively.

A water cooling system was attached at the top and bottom of the chamber for the purpose of compensating variation of the cavity length caused by a seasonal temperature change in the ATF environment. Fig. 3.16 shows the top view of the vacuum chamber. Brown line in Fig. 3.17 shows the air temperature in the ATF-DR room. Other colored lines in Fig. 3.17 show the temperature out side of the vacuum chamber.
Region of blue arrow lines shows the period when the Compton chamber cooled by cooling system. The repetition rate where laser pulses, which is inversely proportional to the length of the cavity, is shown by the red arrows. From Fig. 3.17, we see that variation of the repetition rate of the laser pulse, i.e., length of the cavity, can be compensated by controlling temperature of the vacuum chamber with the water cooling system.

Figure 3.15: The vacuum chamber (enclosed by the box) in the electron beam line. The vacuum chamber is loaded on the optical table. The optical table is loaded on the movable table. The red line shows the electron beam line.

Figure 3.16: The top view of vacuum chamber. Copper colored plate is the water cooling plate which is made by copper for good thermal conductivity.
3.3 Feedback system

3.3.1 Getting an error signal by the tilt lock method

We adopted tilt locking technique for resonance control (§2.5.2.1). The reflection light is monitored by a split photo diode as shown in Fig. 3.10. The split photo diode is used for making the error signal because intensity difference of left side and right side (or upper and lower) is used. Fig. 3.18 shows the error signal of the tilt locking technique. The yellow line shows the intensity of the transmitted light of the optical resonant cavity, the purple line shows the error signal of the tilt locking technique.

3.3.2 Requirement of feedback

The lengths the optical resonant cavity and the laser oscillator cavity must be kept the same as described in §2.4.1. Laser pulses and electron bunches must be synchronized each other. Because the repetition rate of the laser pulses is determined by the length of the laser oscillator cavity, we must achieve two independent condition simultaneously;

- The length of the optical resonant cavity and the laser oscillator cavity are kept same.

Figure 3.17: The variation of the temperature and the cavity length. Brown line shows the air temperature in the ATF-DR room. Other colored line show the temperature out side of the vacuum chamber. The length of the optical resonant cavity is written at some points.
• The length of the cavities, (and is the repetition rate of the laser pulse), must be matched with the those of electron bunch.

Since the electron timing was given as the RF signal from the ATF, a control system of the laser and the cavity was constructed to achieve requirements.

In the optical resonant cavity, tolerance of the cavity length is about 0.7 nm to keep laser intensity higher than a half of the maximum, which is estimated by the measured finesse and the FSR of the cavity. On the other hand, the tolerance of the timing is not tight. An effective length of the laser pulse can be estimated as $\sqrt{(\text{pulse length})^2 + (\text{timing jitter})^2}$. When the timing jitter and the pulse length are 5 ps (corresponds to 0.8 mm), the effective pulse length is about 7.1 ps which is sufficiently smaller than the electron bunch length is 25 ps.

### 3.3.3 The feedback system

The scheme of the feedback system is shown Fig. 3.19. The features of the scheme are;

• A closed loop is configured with two kind of controls, the resonance and the timing control.
• The error signal from the optical resonant cavity is fed to the laser oscillator cavity while the error signal mode with the laser oscillator cavity and the timing reference is fed to the optical resonant cavity.
The second one is adopted because the resonance control need faster
response so that the error signal should be fed to the laser oscillator
cavity which can respond faster than the optical resonant cavity, while
the timing control allows slower time response which the optical resonant
cavity can follows \[17\].

![Diagram of feedback system for locking pulse timing and keeping
resonance of the optical resonant cavity. The red box shows the laser oscillator. The
blue box shows the optical resonant cavity. Red lines show signal based on laser pulse.
Black lines show the electrical signal.](image)

Fig. 3.21 shows a performance of the feedback system. The yellow line
shows the intensity of the transmitted light of the optical resonant cavity.
The red line shows timing difference of laser pulse. Left figure shows the
case when the length of the optical resonant cavity was scanned without
locking the timing between the laser pulse and electron bunch. Right
figure shows the case when the feedback and timing locking system was
turned on. We achieved to stack laser pulses of 500 ± 2 W in the optical
resonant cavity and to lock laser pulse timing with the jitter of 5 ps.

3.4 Detection system for gamma rays

3.4.1 Gamma ray detector

Fig. 3.21 shows the configuration of the gamma ray detector. The
gamma ray detector consists of a photo multiplier tube (PMT), a col-
ored glass, a light guide and a pure cesium iodide (CsI) crystal. The
photomultiplier tube is R4275-02 (Hamamatsu Photonics LTD.) with di-
ameter of the photocathode of 46 mm \[15\]. The CsI crystal is 70 mm
CHAPTER 3. EXPERIMENTAL SETUP OF GAMMA RAY GENERATION WITH THE OPTICAL RESONANT CAVITY

Figure 3.20: Free run condition and controlled condition. Yellow lines show the intensity of the transmitted light. Red lines shows the phase difference of laser pulse and electron RF signal. Left figure shows scanning cavity length and unlocked timing. Right figure shows keeping resonance of the optical resonant cavity and locking timing.

The CsI has luminescence of two decay components. The decay constant and wavelength of the peak yield for the faster decay component is 2-20 ns and 305 nm, while those for the slower decay component is 100-4000 ns and 450 nm respectively [49].

The colored glass was inserted between the CsI crystal and the photocathode of the PMT to block the slower component from the CsI. Thus the detector has capability to discriminate gamma ray from every turn in the ATF ring while gamma rays from a train (consisting of multiple bunches) piled up in the detector. All surfaces of the CsI, light guide and colored glass were polished and outer surfaces was wrapped with the Teflon tape to improve efficiency of luminescence light propagation to the photocathode. The contact surface between the CsI, light guide and colored glass and photocathode were glued by optical bond to reduce the reflection at the boundary.

Figure 3.21: The configuration of gamma ray detector. The optical filter is put between PMT and CsI crystal.

The gamma ray detector was placed at 18 m down stream of the Compton interaction point. The aperture of the gamma ray detector is defined $2\theta = 5.3$ mrad by the slit placed 9.4 m the interaction point which discriminates energy of the gamma rays at the detector. Fig. 3.22 shows en-
Energy distribution of generated gamma rays by the simulation code CAIN [44]. Left one shows energy distribution of all gamma rays. Right one shows the energy distribution of gamma rays in the aperture. It is found that 33\% of generated gamma rays reach the detector and the energy range of of 19 to 28 MeV with the average of 24.5 MeV.

3.4.1.1 Movable collimator

In front of the gamma ray detector, two lead blocks were placed as shown in Fig. 3.23. The lead block of far side from the gamma ray detector has the hole of 15 mm in diameter. The lead block of near side of the gamma ray detector has the hole of 5 mm in diameter and can be moved in transverse direction remotely while the 15 mm block was placed at fixed position throughout the experiment. The movable block was usually out of the detector aperture unless specified.
3.4.2 Data acquisition system

The signal from the detector is taken by a data acquisition system shown Fig. 3.24. The ADC integrates the signal during the temporal width of the gate provided externally. The timing of the gate for the ADC is provided by the clock which is synchronized with the ATF-DR revolution of 2.16 MHz. The data acquisition system is constructed based on the CAMAC standard modules. The ADC (C009) is 12 bit charge integrate type. We use CCP-F as CAMAC crate controller. The system is capable of taking data up to the rate of 50 Hz.

At the same time with taking data of the gamma ray detector, the transmitted intensity of the optical resonant cavity and phase difference of electron bunch and laser pulses are taken by the ADC.

The DCCT value of the ATF-DR and position of the movable table are also recorded.

![Diagram](image)

Figure 3.24: Data acquisition system for gamma ray detector.

3.4.3 Calibration of gamma ray detector

The gamma ray detector was calibrated by using the cosmic ray. At sea level, the dominant component of the cosmic ray is relativistic muons and can be regarded as Minimum Ionizing Particles (MIPs) [3, pp. 246]. The mean energy loss of muon is \( \frac{dE}{dx}=5.6 \text{ MeV/cm} \) in pure CsI [3, pp. 276]. Thus, energy deposit of a cosmic rays muon is 39.2 MeV when it passes through 70 mm pure CsI. For the energy calibration, 2 plastic
3.4. DETECTION SYSTEM FOR GAMMA RAYS

Scintillation counters are placed top and underneath the pure CsI crystal as shown in Fig. 3.25. Size of the plastic scintillation counter top is 75 mm × 40 mm. One at underneath is 105 mm × 50 mm. The 2 plastic scintillators are set 150 mm apart and 2 plastic scintillators are placed approximately 60 mm from the far end of the CsI crystal. Coincidence of 2 plastic scintillation counters provides trigger for the ADC gate for the cosmic ray signal. Energy deposit of cosmic ray muons is shown in Fig. 3.26. Fig. 3.26 was fitted by Landau distribution. 39.2 MeV was defined to be 127.8 channel by the fitting parameter. A pedestal of ADC was 59.5 channel. Calibration factor from channel to energy was defined as;

\[ E[\text{MeV}] = 0.5857 \times (\text{ADC[ch]} - 59.5) \]

Systematic error of number of gamma rays was estimated 3 % from fitting error.

![Figure 3.25: 2 plastic scintillation counters](image)

![Figure 3.26: Energy deposit of cosmic ray](image)

**Table 3.4** summarizes parameters of the gamma ray detector.

3.4.3.1 Calibration of stacking laser power

We measure the transmitted laser power for the estimation of the stacking laser power in the optical resonant cavity. The setup of calibration for the transmitted laser power is shown in Fig. 3.27. Fig. 3.27 is a magnification of the transmission side of the optical resonant cavity. The transmitted power from the optical resonant cavity is divided by the beam splitter. One is measured by the power meter, and other part is measured by the photo diode. Signal of the photo diode is recorded by the
CHAPTER 3. EXPERIMENTAL SETUP OF GAMMA RAY GENERATION WITH THE OPTICAL RESONANT CAVITY

Table 3.4: Parameters of the gamma ray detector.

<table>
<thead>
<tr>
<th>gamma ray detector</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>energy in aperture</td>
<td>24.5 MeV (19~28 MeV)</td>
</tr>
<tr>
<td>scintillator</td>
<td>cesium iodide</td>
</tr>
<tr>
<td>scintillator size</td>
<td>70 mm × 70 mm × 300 mm</td>
</tr>
<tr>
<td>decay constant</td>
<td>2-20 ns(305 nm) &amp; 100-4000 ns(450 nm)</td>
</tr>
<tr>
<td>optical filter</td>
<td>UV pass</td>
</tr>
<tr>
<td>PMT</td>
<td>R4275-02</td>
</tr>
<tr>
<td>acceptance surface of PMT</td>
<td>φ46 mm</td>
</tr>
<tr>
<td>applied voltage of PMT</td>
<td>2.2 kV (rated voltage 2.7 kV)</td>
</tr>
</tbody>
</table>

ADC. A ADC distribution of the photo diode signal is shown Fig. 3.28 where transmitted power was estimated at 1.572 W. Pedestal of ADC was 70 channel. Using the transmission of the mirror of 0.336 %, calibration factor from channel of the PD signal to the stacking laser power in the cavity was estimated as;

\[
I[W] = 0.8205 \times (\text{ADC}[\text{ch}] - 70) \frac{1}{0.00336}
\]

Systematic error of stacking laser power was estimated 14 % from fluctuation of the power meter by the visual contact.

3.4.3.2 Calibration of laser pulse timing

We monitor the phase difference of the electron beam RF signal and the laser pulse signal as the laser pulse timing information. We used the phase detector of a sawtooth wave output type for feedback as shown in Fig. 3.20, while we used the phase detector of two outputs type (PSD-11
When the phase difference of the electron beam RF signal and the laser pulse signal is $\Delta \phi$, one output form phase detector for monitoring is arc cosine $\Delta \phi$. The other output is arc sine $\Delta \phi$. The phase difference can be estimated from arc cosine $\Delta \phi$ and arc sine $\Delta \phi$. Two outputs are put into the ADC for recording the laser pulse timing. Fig. 3.29 shows the scatter plot of arc cosine $\Delta \phi$ and arc sine $\Delta \phi$ as the laser pulse timing. Horizontal axis is arc cosine $\Delta \phi$ of the laser pulse timing. Vertical axis is arc sine $\Delta \phi$ of the laser pulse timing. The center of the scatter plot was calculated $(x, y) = (1154, 1039)$ by distribution of circle. The laser pulse timing was calculated as:

$$\Delta \phi = \arctan \left( \frac{\arcsin \Delta \phi - 1039}{\arccos \Delta \phi - 1154} \right) (\arccos \Delta \phi \neq 0)$$
Chapter 4

Experiment of Gamma Ray Generation with the Optical Resonant Cavity

4.1 Overview of the experimental procedure

Experiments were performed with 1, 5, 10 and 15 bunches / train with the storage mode described in § 3.1.

First, the vertical and the horizontal position of the optical table relative to the electron beam were scanned to find the optimum interaction position by maximizing number of gamma rays in the detector with 15 bunches operation. Other bunch operations, only vertical position was scanned for the optimum interaction position. For all measurements in a position, the data taking was started and stopped at the same beam currents to normalize the number of gamma rays. The timing of the laser pulse was scanned to find the optimum interaction timing. After finding the optimum interaction point, the movable collimater was inserted. The position of movable collimater was scanned to confirm whether the line between the interaction point and the detector is within the aperture of the lead shield placed between. After that, we took data for statistics at the optimum position. The beam current of the ATF-DR during the data taking was recorded by the DCCT.

4.2 Experimental result

4.2.1 Results of 15 bunches operation

Fig. 4.1 shows number of gamma rays versus the vertical position of the optical table. We found the vertical position of the optical table was $-0.684$ mm from the mean of Gaussian fitting. Fig. 4.2 shows number of gamma rays versus laser pulse timing scanning. The laser pulse timing was set to 0.308 ns.
CHAPTER 4. EXPERIMENT OF GAMMA RAY GENERATION WITH THE OPTICAL RESONANT CAVITY

Figure 4.1: Vertical position scanning with 15 bunches operation.  

Figure 4.2: Laser pulse scanning with 15 bunches operation.

Fig. 4.3 and Fig. 4.4 show number of gamma rays versus vertical and horizontal position of the movable collimator. From the position of the collimator, we confirmed the detector is aligned to the electron beam axis.

Figure 4.3: Vertical scanning of the movable collimator position with 15 bunches operation.  

Figure 4.4: Horizontal scanning of the movable collimator position with 15 bunches operation.

A typical waveform of the WCM was shown in Fig. 4.5 with a 15 bunch operation. A distribution of the transmitted laser power is shown Fig. 4.6. The mean of the transmitted laser power was $1.674 \pm 0.282$ W. The error was calculated by RMS/Entries of the distribution. Stacking laser power was estimated to be:

$$\frac{1.674 \pm 0.282W}{0.00336} = 498 \pm 2 \text{ W}$$

A distribution of laser pulse timing is shown Fig. 4.7. Horizontal axis in Fig. 4.7 is the relative laser pulse timing with RF frequency. The timing jitter calculated by RMS of distribution in Fig. 4.7 was estimated 5 ps.

Fig. 4.8 shows the energy deposit of gamma rays in the detector with
15 bunch operation. The average energy of gamma rays was estimated to be 24.5 MeV by the simulation code CAIN (§3.4.1). The mean energy deposit in Fig. 4.8 is 636.0 ± 3.1 MeV by Gaussian fitting. Therefore, observed number of gamma ray was 26.0 ± 0.1 gamma rays per laser-electron beam train crossing with the 15 bunch operation. It correspond to \((1.70 \pm 0.07) \times 10^8\) gamma rays per second in all solid angles (§3.4.1) because revolution frequency of the ATF-DR is 2.16 MHz.

### 4.2.2 Results of each bunch

The experimental results are summarized in Table 4.1 for each condition. “e\(^{-}\) current” shows the electron beam current in the ATF-DR. The electron beam current during the data taking was estimated from the current at the beginning and the end of the date taking assuming that the beam current decrease linearly in time. “Laser power” is the estimated stacking laser power. “Normalized” is the number of gamma
CHAPTER 4. EXPERIMENT OF GAMMA RAY GENERATION WITH THE OPTICAL RESONANT CAVITY

Figure 4.8: Energy deposit of generated gamma ray in the gamma ray detector with 15 bunches operation.

rays normalized by the stacking laser power and electron beam current. It is calculated as:

\[
\text{Number of gamma rays [1/train]} = \frac{\text{electron beam current [A]} \cdot \text{stacking laser power [W]}}{\text{Energy deposit of generated gamma ray}}.
\]

“Simulation” is the expected number of gamma ray generation estimated by CAIN. The electron beam size was estimated to be 170 µm (horizontal) \(\times\) 12 µm (vertical) by the computer code SAD with the measured emittance and the estimated \(\beta\) function into account. The error of the emittance was 50%. The uncertainty of electron bunch length was main source of the ambiguity of the estimate, and was changed from 6.0 mm to 9.0 mm by the conditions of the accelerator. There were additional 3 % systematic errors common for all number of gamma rays measurements (see 3.4.3). There were additional 14 % systematic errors common for all Laser power measurements (see 3.4.3.1).

4.2.3 Discussions

The number of gamma rays was consistent with the simulation at smaller bunch operation, the number of gamma rays was getting smaller than the simulation for larger bunch operation. The reason for these deviations is under investigation. We suspect an effect of the synchrotron oscillation. We monitored bunch length by the streak camera at the
Table 4.1: Summary of experimental results and conditions. There are additional 3\% systematic errors common for all number of gamma ray measurements. There are additional 14\% systematic errors common for all Laser power measurements.

<table>
<thead>
<tr>
<th>Number of bunches</th>
<th>$e^-$ current [mA]</th>
<th>Laser power [W]</th>
<th>Number of gamma rays [1/train]</th>
<th>Simulation</th>
<th>Normalized [gamma/A W]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.2</td>
<td>437 ± 2</td>
<td>5.4 ± 0.3</td>
<td>4.9 ± 0.3</td>
<td>5.6 ± 0.3</td>
</tr>
<tr>
<td>5</td>
<td>4.7</td>
<td>423 ± 2</td>
<td>10.6 ± 0.1</td>
<td>10.5 ± 0.5</td>
<td>5.3 ± 0.1</td>
</tr>
<tr>
<td>10</td>
<td>8.5</td>
<td>470 ± 2</td>
<td>19.0 ± 0.1</td>
<td>21 ± 1</td>
<td>4.8 ± 0.1</td>
</tr>
<tr>
<td>15</td>
<td>11</td>
<td>498 ± 2</td>
<td>26.0 ± 0.1</td>
<td>29 ± 1</td>
<td>4.8 ± 0.1</td>
</tr>
</tbody>
</table>

ATF-DR. In Fig. 4.9 shows long-range view of the streak camera. From Fig. 4.9, we see that the oscillation of the longitudinal direction increases the effective bunch length on average. An effective bunch length including bunch oscillation could be defined as shown in Fig. 4.10. Fig. 4.11 shows the effective bunch length of the last bunch in a train versus bunch current. We see that the effective bunch length tends to be longer for higher electron beam current which could be a reason our observation.

The experiment indicates necessity of a bunch by bunch information of photon generation since stable photon yield is required for the ILC positron source based on laser Compton scattering.

Figure 4.9: Image of the streak camera. The oscillation of the longitudinal direction increases the effective bunch length on average.
Figure 4.10: The effective bunch length of the last bunch in a train versus bunch current. The effective bunch length tends to be longer for higher electron beam current which could be a reason our observation.
Chapter 5

Improved Experimental Setup of Bunch by Bunch Measurement with Higher Finesse Cavity

Based on the successful generation and detection of gamma rays by the laser-Compton scattering at the ATF. Further improvements were planned, i.e., increasing number of photons by more laser power stacking in the cavity as well as improvement of detector system which has capable of detecting bunch by bunch photon yield. As the work is in progress, a status and preliminary results of the improvement are described in this chapter.

5.1 Increasing enhancement factor of the optical resonant cavity

5.1.1 Changing to high reflectivity mirror

The structure of the optical resonant cavity is the same as described in chapter 3. We increase enhancement factor of the cavity from 250 to 760 by changing the reflectivity of the mirror of the transmission side from 99.64 % to 99.94 %, where the reflectivity was estimated by measured finesse of the cavity (§ 5.2.1). The transmittance of the mirror was 0.023 %, which was the ratio of the measured incident power and the measured transmission power.

5.1.2 Modification of the optical resonant cavity

A large variation of the cavity length due to seasonal temperature change in the ATF environment can be compensated as described in § 3.2.4. Thus, the long piezo (see § 3.2.2) was replaced with the shorter
one and the holder of its actuator was changed accordingly.

5.2 Improved optical system and feedback system for higher finesse cavity

5.2.1 Modification of the laser optics

After the modification described in previous section, we observed an irregular behavior in the phase difference of the laser pulse and the electron RF signal as shown in Fig. 5.1 when the length of the cavity was scanned.

Figure 5.1: The strange behavior of the phase difference signal of the laser pulse.

We observed three phenomena;

- The phase difference signal was stable when the laser beam was dumped on the laser path.
- The phase difference signal was stable when the length of the cavity was not scanned.
- The phase difference signal was unstable when the cavity resonated.

We deduced that the reflected light from the optical resonant cavity disturbed the laser oscillator. To block the reflected light, an isolator was added in the optical path. Arrangement of laser optical components was changed as shown Fig. 5.2. In consequence, we stabilized the phase difference signal by reducing the reflected light with the isolator.

Fig. 5.2 shows the intensity of the transmitted light of the optical resonant cavity when the length of the optical resonant cavity was scanned. The measured finesse is about 1800 and estimated waist size is about 60 μm (2σ). The stacking power of the peak is about 1.5 kW as achieved.
5.2. IMPROVED OPTICAL SYSTEM AND FEEDBACK SYSTEM FOR HIGHER FINESSE CAVITY

Figure 5.2: Modified laser optics. The isolator was added in the optical path.

Figure 5.3: The intensity of the transmitted light with scanning the length of the optical resonant cavity.

5.2.2 Modification of the feedback system

Since the enhancement factor was increased and resonance width became narrower. Faster control of the cavity length was required to keep the cavity on resonance. The performance of the feedback system described in § 3.3.3 is shown in Fig. 5.4. It is seen from the figure that the timing of laser pulse and electron bunch (which was measured as phase difference) fluctuate 150 ps in the RMS, when the cavity length was locked to the laser as shown Fig. 5.4. It is larger than electron bunch length 25 ps.

Thus the feedback system was improved as Fig. 5.5. We added signal
CHAPTER 5. IMPROVED EXPERIMENTAL SETUP OF BUNCH BY BUNCH MEASUREMENT WITH HIGHER FINESSE CAVITY

Figure 5.4: The response of each signal with previous feedback system. Yellow line shows the transmitted light of the optical resonant cavity. Blue line shows the error signal for tilt locking technique. Red line shows the phase difference of the laser pulse timing.

Figure 5.5: The scheme of the feedback system for locking pulse timing and keeping resonance of the optical resonant cavity. Red box shows the laser oscillator. Blue box shows the optical resonant cavity. Red lines show signal based on laser pulse. Black lines show electrical signal.

In Fig. 5.6, the red line shows the intensity of the transmitted light of the optical resonant cavity. The blue line shows the timing difference of laser pulse. Left figure shows scanning the optical resonant cavity length and unlocked timing. Right figure shows performance of the feedback system when the feedback system was turned on. We achieved to stack laser pulses of 1500 W in the optical resonant cavity and to lock the laser pulse timing within 2 ps to the election bunches. Table 5.1 summarizes parameters of the optical resonant cavity.
5.3. EXPERIMENT OF GAMMA RAY GENERATION WITH HIGHER FINESSE CAVITY

Experiments were done with 1 and 10 bunches / train with the higher finesse cavity. The overall procedure was described in chapter 4.

Results of the experiment are summarized in Table 5.2 for each condition. The number of gamma rays to the electron beam current increases than previous measurement (Table 4.1). The number of gamma rays were not proportional to the stacked laser power. It is because the ATF parameters were not optimized for laser Compton experiment, so that the electron beam size was larger than the optimum in this experiment.

![Graph](image)

Figure 5.6: Free run condition and controlled condition. Red lines show intensity of the transmitted light. Blue lines shows phase difference of laser pulse and electron RF signal. Left figure show scanning cavity length and unlocked timing. Right figure shows keeping resonance of the optical resonant cavity and locking timing.

### Table 5.1: Parameters of the optical resonant cavity.

<table>
<thead>
<tr>
<th>optical resonant cavity</th>
<th>type</th>
<th>2mirror Fabry-Perot</th>
</tr>
</thead>
<tbody>
<tr>
<td>injection of mirror</td>
<td>R=99.64 %, T=0.336 %</td>
<td></td>
</tr>
<tr>
<td>transmission of mirror</td>
<td>R=99.94 %, T=0.023 %</td>
<td></td>
</tr>
<tr>
<td>finesse</td>
<td>1800</td>
<td></td>
</tr>
<tr>
<td>enhancement</td>
<td>760</td>
<td></td>
</tr>
<tr>
<td>waist size</td>
<td>60 µm (2σ)</td>
<td></td>
</tr>
<tr>
<td>cavity length</td>
<td>420 mm</td>
<td></td>
</tr>
<tr>
<td>stacking intensity</td>
<td>1 ~ 1.5 kW</td>
<td></td>
</tr>
<tr>
<td>timing jitter of laser pulse</td>
<td>2 ps</td>
<td></td>
</tr>
</tbody>
</table>

5.3 Experiment of Gamma Ray Generation with Higher Finesse Cavity

Experiments were done with 1 and 10 bunches / train with the higher finesse cavity. The overall procedure was described in chapter 4.
Table 5.2: Summary of experimental results and conditions.

<table>
<thead>
<tr>
<th>Number of bunches</th>
<th>$e^-$ current [mA]</th>
<th>Laser power [kW]</th>
<th>Number of gamma rays [1/train]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.2</td>
<td>1.48 ± 0.04</td>
<td>10.8 ± 0.1</td>
</tr>
<tr>
<td>10</td>
<td>6.7</td>
<td>1.48 ± 0.06</td>
<td>26.8 ± 0.1</td>
</tr>
</tbody>
</table>

5.4 Detection system for bunch by bunch gamma ray yield

5.4.1 Modification of KEK-ATF

After the measurement of §5.3, repetition frequency of the electron bunch was changed from 357 MHz to 178.5 MHz for strip line kicker study et.c. [48]. Thus bunch spacing of the KEK-ATF was changed from 2.8 ns to 5.6 ns. RF gun was improved from 1.6 cell to 3.6 cell one and was expected to improve the phase stability of the electron beam [49].

5.4.2 Gamma ray detector

The new gamma ray detector consists of a photo multiplier tube (PMT), an optical band pass filter and two barium fluoride (BaF$_2$) crystals. The photomultiplier tube is H3378-51 based on R3377 (Hamamatsu Photonics LTD.) with diameter of the photocathode of a 46 mm [48]. The rise time of H3378-51 is about 0.7 ns with the nominal voltage 3.0 kV. BaF$_2$ crystal has two luminescence components [3, pp. 276]. Each luminescence rate are higher than CsI luminescence rate. The decay constant and peak wavelength of the fast decay component is 0.9 ns and 220 nm while those of the slow decay component is 630 ns and 300 nm respectively. The decay constant is faster than electron bunch spacing 5.6 ns if the slow decay component is removed by a optical filter. Fig. 5.7 shows BaF$_2$ luminescence spectrum. We inserted a optical band pass filter between BaF$_2$ crystal and PMT for reducing the slow decay component. The band width of the filter is 220 ± 10 nm and the transmission of the filter is about 20 % at 220 nm [50].

BaF$_2$ crystals are 70 mm wide, 70 mm high and 100 mm long [51]. The radiation length of BaF$_2$ is 20.3 mm. We connected two crystals to longitudinal direction. The crystal becomes 200 mm long which is 9.85 radiation length [47]. Crystals were coupled with silicon grease TSF451-

*1 The gamma ray detector is shared with the Laser Wire beam size monitor system (LW system)
5.4. DETECTION SYSTEM FOR BUNCH BY BUNCH GAMMA RAY YIELD

50M. Transmittance of 12 \( \mu \text{m} \) thickness grease is more than 90% at \( \lambda = 220 \text{ nm} \) [50, 51, 52].

All surfaces were polished for increasing transmittance to the adjoined crystal. The outside on all surfaces were wrapped with black collared paper for suppression of the reflection of the luminescence. Fig. 5.8 shows the configuration of fast gamma ray detector.

The CsI detector was replaced with the new one with the BaF\(_2\) and set 18 m downstream of the Compton interaction point. It had the same aperture of \( 2\theta = 5.3 \text{ mrad} \) with the previous setup.

5.4.2.1 Simulation of energy deposit

The energy deposit of gamma rays to the detector was calculated by Geant4 [53]. The energy distribution of gamma rays in the detector is determined by the aperture of the slit described as shown in Fig. 5.9.

---

\[\text{LW system require the length of the scintillator crystal which is about 10 radiation length or more for separating Compton gamma rays from bremsstrahlung photons.}\]
Gamma rays were injected normal to the crystal at its center. Crystal size is 70mm \times 70mm \times 200mm. By the simulation, average of energy deposit was find to be 21.17 MeV.

![Simulation of energy deposit in 70mm \times 70mm \times 200mm BaF\textsubscript{2} crystal.](Image)

**Figure 5.9:** Simulation of energy deposit in 70mm \times 70mm \times 200mm BaF\textsubscript{2} crystal.

### 5.4.3 Data acquisition system

The signal of the gamma ray detector is taken by the data acquisition system shown in Fig. 5.10. A digital oscilloscope takes the waveform of the gamma ray detector. In order to suppress the loss of high frequency components in the signal, the digital oscilloscope is placed near the gamma ray detector. The digital oscilloscope is DSO5054a (Agilent Technologies Japan, Ltd.) [64]. Bandwidth of DSO5054a is 500 MHz and the maximum sampling rate is 4 Giga-Sample/second. The ADC takes the charge of the signal for counting number of gamma rays in single train. Gate for the ADC and the trigger for the oscilloscope is made by the clock from the ATF-DR revolution 2.16 MHz. ADC is C009 as 12 bit charge integrate type for CAMAC [50]. We use CC/NET as CAMAC crate controller [65]. At the same time with taking data of the gamma ray detector, the transmitted intensity of the optical resonant cavity and the phase difference of electron bunches and laser pulses are recorded by the ADC. The beam current of the ATF-DR and the position of the movable table are recorded during the data acquisition by the EPICS system [66].
5.4. DETECTION SYSTEM FOR BUNCH BY BUNCH GAMMA RAY YIELD

Figure 5.10: Data acquisition system for the gamma ray detector. The signal of detector is divided for taking waveform and counting number of gamma rays in single train.

5.4.4 Calibration of gamma ray detector

5.4.4.1 Optimization of applied voltage of the PMT

In the experiment, the revolution frequency of the electron beam is 2.16 MHz and Compton gamma rays are generated with 2.16 MHz. With this condition, the PMT could be saturated if it was operated at the nominal voltage. In Fig. 5.11, number of gamma rays at the timing scan with the applied voltage to the PMT of 2.1 kV is shown. Fig. 5.12 is the same plot but with the voltage of 1.4 kV. We observed that the PMT was saturated with 2.1 kV but was not saturated with 1.4 kV. Therefore, applied voltage of PMT is set to be 1.4 kV.
5.4.4.2 Calibration of output voltage of gamma ray detector

Gamma ray detector is calibrated by cosmic rays. The energy loss of cosmic rays is 6.6 MeV/cm in BaF$_2$ [3, pp. 276]. The energy deposit of muons is 46.2 MeV in 70 mm BaF$_2$. For calibration, 2 plastic scintillation counters are placed top and under the BaF$_2$ crystal. Coincidence of 2 plastic scintillation counters provides trigger in Fig. 5.11 for the ADC gate signal for the cosmic ray signal. The setup of detecting cosmic rays is the same as Fig. 3.25. Fig. 5.13 shows a output voltage of the gamma ray detector with cosmic rays with the digital oscilloscope. The output voltage was defined the voltage of peak to peak on the digital oscilloscope with trigger signal. As the gain of the PMT increases by power of the applied voltage. Fig. 5.13 was fitted by $v_n = P_0x^{P_1} + P_2$ to find the output voltage at 1.4kV, where x was the applied voltage and $P_0$, $P_1$, $P_2$ are fitting parameter. By the fitting parameters. 1 gamma ray (21.17 MeV) is defined $k_g = 1.03 \pm 0.11$ mV. Measured rise time of PMT was 1.4 ns with 1.4 kV. We expect that information of multi bunches can be separated because it is faster than the electron bunch spacing 5.6 ns. Table 5.3 summarizes parameters of gamma ray detector.
5.4. DETECTION SYSTEM FOR BUNCH BY BUNCH GAMMA RAY YIELD

Figure 5.13: Output voltage of gamma ray detector by cosmic ray.

Table 5.3: Parameters of the gamma ray detector.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>scintillator</td>
<td>barium fluoride</td>
</tr>
<tr>
<td>scintillator size</td>
<td>70 mm × 70 mm × 200 mm</td>
</tr>
<tr>
<td>energy deposit in aperture</td>
<td>21.17MeV (19~28 MeV)</td>
</tr>
<tr>
<td>decay constant</td>
<td>0.9 ns(220 nm) &amp; 630 ns(300 nm)</td>
</tr>
<tr>
<td>bandwidth of optical filter</td>
<td>220 ± 10 nm</td>
</tr>
<tr>
<td>PMT</td>
<td>H3378-51 based on R3377</td>
</tr>
<tr>
<td>acceptance surface of PMT</td>
<td>φ46 mm</td>
</tr>
<tr>
<td>applied voltage of PMT</td>
<td>1.4 kV</td>
</tr>
<tr>
<td>measured rise time of PMT</td>
<td>1.4 ns</td>
</tr>
</tbody>
</table>
Chapter 6

Experiment of Bunch by Bunch Measurement with Higher Finesse Cavity

6.1 Overview of the experimental procedure

Experiments were performed with 1 and 10 bunch operation. The operation modes of the ATF was described in §3.1. Data was taken with the storage mode.

First, position of the optical table relative to the electron beam was scanned for optimum interaction position by maximizing energy deposit of gamma rays in the detector with 15 bunch operation. After finding the optimum position, we took waveform of the gamma ray detector. Then the beam current of the ATF-DR was recorded by the DCCT. Fig. 6.1 shows the procedure for taking waveform data. The electron beam was refilled when the beam current in the ATF-DR was getting low and the data was recorded until sufficient number of waveforms were accumulated for statistic.

The waveforms of the WCM were recorded at the beginning and the end of the ADC data taking. The applied voltage to the gamma ray detector was 1.4 kV as we defined the optimum applied voltage in §5.4.4.1.

6.2 Experimental results

6.2.1 Results of 1 bunch operation

Fig. 6.2 shows gamma ray yield when scanning vertical position of the optical table. Fig. 6.3 shows results for the laser timing scan. The gamma ray yield was normalized by the DCCT value. We decided the vertical position of the optical table was $-0.283$ mm and laser pulse timing was
CHAPTER 6. EXPERIMENT OF BUNCH BY BUNCH MEASUREMENT WITH HIGHER FINESSE CAVITY

Taking waveform & Writing DCCT value
Taking WCM & Writing DCCT value

Figure 6.1: Procedure of taking waveform.

4.39 rad by the mean of Gaussian fitting for each scan, respectively.

Once the table position and the laser pulse timing were fixed, we took 53 waveforms. A typical waveform of the gamma ray detector and the WCM were shown in Fig. 6.2 and Fig. 6.3 respectively.

Figure 6.2: Vertical position scanning with 1 bunch operation.
Figure 6.3: Laser pulse scanning with 1 bunch operation.

6.2.2 Results of 10 bunch operation

Same as the 1 bunch operation, Fig. 6.4 shows results for the vertical position scan of the optical table. Fig. 6.5 shows results for the laser timing scan. We decided the vertical position of the optical table was

Figure 6.4: Vertical position scanning with 10 bunch operation.
Figure 6.5: Laser pulse scanning with 10 bunch operation.
6.3 Analysis

6.3.1 Beam current analysis

The DCCT measure the beam current of the train while the WCM shows ratio of each bunch current but does not measure the absolute value of bunch current. We calculated each bunch current from the product of the ratio of each bunch current and the beam current of the train.
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Fig. 6.8: Waveform of the gamma ray detector with 10 bunch operation.

Fig. 6.9: Waveform of the WCM with 10 bunch operation.

Fig. 6.10 shows bunch current of each bunch for a 10 bunch operation. It showed that the bunch current tended to lower for larger bunch number (tail of a train).

Fig. 6.10: The bunch current of each bunches with 10 bunches operation.

6.3.2 Consideration of fitting function for waveform

The signal of the gamma ray detector consists; the propagation light in the scintillator crystal, transmission and the rise time in the PMT; and the decay time of the scintillator crystal. For one decay time component, we assume that the signal of the gamma ray detector is convolution of a Gaussian and an exponential function as follows;
6.3. ANALYSIS

\[ S(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}\sigma} \exp \left( -\frac{x^2}{2\sigma^2} \right) \frac{Q}{\tau} \exp \left( -\frac{t-x}{\tau} \right) \text{step-function}(t-x) dx \]

\[ = \frac{Q}{2\tau} \exp \left( \frac{\sigma^2}{2\tau^2} - \frac{t}{\tau} \right) \text{erfc} \left( \frac{\sigma}{\sqrt{2\tau}} - \frac{t}{\sqrt{2\sigma}} \right). \]

The \( \sigma \) is the smearing of the signal in the detector, including the propagation light in the scintillator crystal, the delay of dynodes and the rise time of the PMT. \( Q \) shows signal charge. \( \tau \) shows time constant of the decay. \( t \) shows time. \( \text{erfc} \) is the complementary error function.

BaF\(_2\) crystal has two decay components (\S\ 5.4.2). We put each signal based on each decay component of BaF\(_2\) crystal as;

Slow : \( S_s(t) = \frac{Q_s}{2\tau_s} \exp \left( \frac{\sigma^2}{2\tau_s^2} - \frac{t-t_0}{\tau_s} \right) \text{erfc} \left( \frac{\sigma}{\sqrt{2\tau_s}} - \frac{t-t_0}{\sqrt{2\sigma}} \right), \)

Fast : \( S_f(t) = \frac{Q_f}{2\tau_f} \exp \left( \frac{\sigma^2}{2\tau_f^2} - \frac{t-t_0}{\tau_f} \right) \text{erfc} \left( \frac{\sigma}{\sqrt{2\tau_f}} - \frac{t-t_0}{\sqrt{2\sigma}} \right). \)

\( S_s(t) \) is for slow decay component of BaF\(_2\) crystal. \( S_f(t) \) is for fast decay component. \( t_0 \) is the time offset. \( \tau_s \) is fixed at 630 ns, slower decay component of BaF\(_2\). Because the PMT was operated at lower voltage of 1.4 kV, the fall time of the signal could be longer so that the decay constant of the fast component \( \tau_f \) was set to a fitting parameter rather than being fixed at the one from the BaF\(_2\) characteristics.

Since we observed an additional component in waveforms, we added an empirical component \( S_m(t) \) as;

Middle : \( S_m(t) = \frac{Q_m}{2\tau_m} \exp \left( \frac{\sigma^2}{2\tau_m^2} - \frac{t-t_0-t_m}{\tau_m} \right) \text{erfc} \left( \frac{\sigma}{\sqrt{2\tau_m}} - \frac{t-t_0-t_m}{\sqrt{2\sigma}} \right). \)

\( t_m \) shows the time offset of middle component. Therefore, we defined a fitting function for waveform of 1 bunch as;

Fitting function : \( S_1(t) + \text{offset} = S_s(t) + S_m(t) + S_f(t) + \text{offset}. \)

Here for identified the decay component range of time constants are limited as;

\[ 0.8 \text{ ns} \leq \tau_f \leq 4 \text{ ns}, \]
\[ 4 \text{ ns} \leq \tau_m \leq 11 \text{ ns}, \]
\[ \tau_s = 630 \text{ ns}. \]
6.3.3 Analysis of 1 bunch waveform

First, waveform of 1 bunch was fitted by all parameters being free. It is referred as the preliminary-fitting 1. In the fitting, \( Q_s \) is assumed to be proportional to \( Q_f \) as \( Q_s = k_{q} Q_f \). Fig. 6.11 shows typical fitted waveform with the preliminary-fitting 1. After fitting all 1 bunch waveforms, \( k_{q}, \tau_{f}, \tau_{m}, \sigma \) and \( t_{m} \) are fixed to the mean of all waveforms. The parameters for preliminary-fitting 1 were; \( Q_s = 3.36325 Q_f, \tau_f = 1.90955, \tau_m = 8.12848, \sigma = 0.80104 \) and \( t_m = 3.94536 \).

After fixing parameters, waveforms were fitted again by \( t_0, Q_f \) and \( Q_m \) being free. It is referred as the preliminary-fitting 2. Fig. 6.12 shows a typical fitted waveform with the preliminary-fitting 2. Fig. 6.13 shows the plot of \( Q_m \) and \( Q_f \). \( Q_m \) shows correlation with \( Q_f \) in Fig. 6.13. From Fig. 6.13, \( Q_m \) was assumed to be a function of \( Q_f \) as; \( Q_m = 0.1409 Q_f \).

![Figure 6.11: Preliminary fitting 1 of waveform with 1 bunch operation.](image1)

![Figure 6.12: Preliminary fitting 2 of waveform with 1 bunch operation.](image2)

Finally, waveforms of 1 bunch data were fitted with \( t_0 \) and \( Q_f \) being free. Fig. 6.14 shows a typical fitted waveform.

6.3.4 Analysis of 10 bunches waveform

Taking bunch spacing of electron bunch of 5.6 ns into account, a fitting function of waveform with 10 bunch operation is defined as;

\[
S_{10}(t) + offset = \sum_{k=0}^{9} S_1(t - 5.6 \text{ ns} \times k) + offset.
\]

Parameters except for \( Q_f, t_0 \) and \( offset \) were fixed at the average of fitting parameter with 1 bunch data.
6.3. ANALYSIS

6.3.5 Number of gamma rays

6.3.5.1 Determination of the calibration factor

We determined the relation of the signal voltage from the gamma ray detector and the energy deposit of the cosmic rays (minimum ionizing particles). The factor, $k_g$, was determined to be $k_g = 1.03$ as described in § 5.4.3.2. While we obtain the information of the signal charge $Q_f$ from the gamma ray detector, the relation of the signal voltage and fitting parameters is needed. We calculated the maximum of $S_1(t)$ numerically.
with fixed parameters. The relation factor \( k_m \) is 1.05279 as;

\[
\text{maximum of } S_1(t) = k_m \frac{Q_f}{2 \tau_f}.
\]

Therefore,

\[
\text{Number of gamma ray} = \frac{k_m Q_f}{k_g 2 \tau_f} = \frac{1.05279}{1.03} \frac{Q_f}{2 \times 1.90955} = (0.268 \pm 0.029) Q_f.
\]

(6.1)

Fig. 6.16 shows a distribution of the number of gamma rays for 1 bunch operation. The distribution was fitted by a Poisson distribution and the mean of gamma rays was 8.17 ± 0.50 /train.
Fig. 6.17 shows a distribution of the number of gamma rays /bunch with the 10 bunch operation. The mean of gamma rays was $5.39 \pm 0.11$ /bunch by the Poisson fitting.

![Figure 6.17: Distribution of the number of gamma rays /bunch with 10 bunch operation.](image)

The number of detected gamma rays was calculated independently by changing the scale factor of the measured charge $Q_f$ to the number of gamma rays. Fig. 6.13 and Fig. 6.14 show the distribution of $Q_f$ for 1 bunch and 10 bunch case. The distributions were fitted with the Poisson function with a scaling factor $k_s$ as:

$$P = \frac{e^{-\lambda}k_s^Q_f}{(k_sQ_f)!},$$

where $\lambda$ is the expected number of gamma rays.

In 1 bunch case, scaling factor was $0.225 \pm 0.084$, and was $0.225 \pm 0.016$ for 10 bunch case. The scaling factors for 1 bunch and 10 bunch case were consistent each other and were also consistent with the factor obtained by cosmic ray calibration ($0.268 \pm 0.029$ Eq. 6.1). We adopt errors and the scaling factor of 10 bunch case by this method because statistics of 10 bunches is larger than statistics of 1 bunch. Thus, the number of detected gamma rays was $6.69 \pm 0.47$ and $4.57 \pm 0.32$ for 1 bunch and 10 bunch operation respectively.

6.3.5.2 Correlation of the number of gamma rays and beam conditions.

Fig. 6.20 shows correlation of the number of gamma rays normalized by the laser power and electron bunch current. The number of gamma rays is estimated from the cosmic ray. We see that the number of gamma
CHAPTER 6. EXPERIMENT OF BUNCH BY BUNCH MEASUREMENT WITH HIGHER FINESSE CAVITY

Figure 6.18: The histogram of $Q_f$ is fitted by scaled Poisson distribution with 1 bunch operation.

Figure 6.19: The histogram of $Q_f$ is fitted by scaled Poisson distribution with 10 bunch operation.

rays is reasonably proportional to the electron bunch current.  
Fig. 6.21 shows correlation of gamma rays normalized by the laser power and the electron bunch current. We see that the number of gamma rays was proportional to the electron bunch for lower current but tended to lose a proportionality for higher current. The number of gamma rays normalized by the laser power and electron current for each bunch for 10 bunch operation is plotted in Fig. 6.22. The red line is the average for all bunches. We did not see appreciable dependence of the number of gamma rays on the position of a bunch in a train. The observation supported enlargement of effective bunch length
discussed in §4.2.3.

Figure 6.20: The scatter plot of laser normalized number of gamma rays and electron bunch current.

Figure 6.21: The scatter plot of laser normalized number of gamma rays and electron bunch current.

6.3.5.3 Absolute yield of gamma rays

The results of the experiment are summarized in Table 6.1 for each conditions. “e− current” shows the current of the electron beam, which was calculated from the mean of DCCT of data taking. “Laser power” is the estimated stacking laser power in the optical resonant cavity. “Number of gamma with conv.” is the number of gamma rays calculated by the conversion factor (see Eq. 6.1). “Number of gamma with Poisson” is the number of gamma rays calculated by the scaling factor (see...
CHAPTER 6. EXPERIMENT OF BUNCH BY BUNCH MEASUREMENT WITH HIGHER FINESSE CAVITY

Figure 6.22: Normalized number of gamma rays each bunch number. Red line is mean of all bunches of all waveforms with 10 bunch operation.

§ 6.3.5.1. “Number of gamma with CAIN” is the expected number of detected gamma rays calculated by the simulation code CAIN [13]. Error of “Number of gamma with CAIN” is calculated by the error of “e− current” and “Laser power”. There are 14% systematic errors for the estimation of the stacking laser power (see 3.4.3.1). There are additional 11% systematic errors for the error of calibration factor $k_g$ for “Number of gamma with conv.” (see 5.4.4.2). There are additional 13% systematic errors from fitting parameters for “Number of gamma with conv.”. Each systematic error was estimated such that; A variation of $Q_f$ was calculated by fitting of waveform with 1 bunch operation when each fitting parameter (offset, $\sigma$, $\tau_f$, $Q_s$, $\tau_m$ and $t_m$) changes ± RMS of the mean and by changing $Q_m$ with its error in Fig. 6.13. We assumed each systematic error for $Q_f$ was added quadratically.

All measurement are consistent with expectations within error. The main source of the uncertainty came from parameters to fix waveform.

Table 6.1: Comparison with experimental results and simulation.

<table>
<thead>
<tr>
<th>Number of bunches</th>
<th>$e^−$ current [mA]</th>
<th>Laser power [kW]</th>
<th>Number of gamma with conv. [1/bunch]</th>
<th>Number of gamma with Poisson [1/bunch]</th>
<th>Number of gamma with CAIN [1/bunch]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.86 ± 0.04</td>
<td>1.10 ± 0.02</td>
<td>8.17 ± 0.50</td>
<td>6.69 ± 0.47</td>
<td>10.3 ± 0.3</td>
</tr>
<tr>
<td>10</td>
<td>9.0 ± 0.2</td>
<td>1.04 ± 0.02</td>
<td>5.39 ± 0.11</td>
<td>4.57 ± 0.32</td>
<td>4.72 ± 0.14</td>
</tr>
</tbody>
</table>
Chapter 7

Conclusion

We studied a gamma ray generation system based on laser Compton scattering for development of the ILC polarized positron source. For increasing number of polarized positron generation, we performed R&D for increasing laser intensity by the optical resonant cavity.

We installed the optical resonant cavity with the length of 420 mm into the KEK-ATF. It was confirmed that the installation of the optical resonant cavity did not affect the operation of the ATF. In the cavity, laser pulses of wavelength of 1064 nm with 357 MHz repetition were stacked and the intensity was enhanced 760 times of the input. In this configuration, we achieved to keep resonance of the optical resonant cavity and to lock the laser pulse timing to the electron bunch. Stacked laser pulse was collided to electron beam with 10 bunch operation. When stacking laser power is $1.04 \pm 0.02$ kW and electron current is 9.0 mA, number of detected gamma ray was $53.9 \pm 1.1$ /train and is corresponded to $(3.53 \pm 0.07) \times 10^8$ gamma rays /second in all solid angles. It was also confirmed that number of gamma ray was consistent with estimate by a simulation.

Toward further development for the ILC positron source, the gamma ray detector was upgraded to measure bunch by bunch gamma ray yield in a train. Preliminary results showed that, the yield measured by the new system consistent with simulation and there were no significant bunch dependent fluctuation for gamma ray yield.

For future development, an improvement of the gamma ray detector is planed to make it possible to operate the PMT with nominal voltage because the detector was not optimized for fast signal detection to avoid the saturation of the PMT under the high repetition data taking. In addition, the data acquisition system for the multi bunch yield measurement is planed. It is replacement of the current waveform recording by the oscilloscope and consist of a front end charge calculation board based
on the system developed at the H1 experiment at HERA. As for the optical resonant cavity, a new twisted 4 mirror cavity is being designed and planned to be install into the ATF-DR in summer 2011.

For conclusion, we successfully demonstrated the gamma ray generation using an optical resonant cavity installed in ultra low emittance accelerator facility. It can be regarded as the possibility to use this technique for polarized positron source as well as usability of the technique to other application.
Appendix A

PDH technique

We will adopt Pound–Drever–Hall (PDH) technique for keeping resonance of the optical resonant cavity [65]. In PDH technique, the input laser beam is modulated by an Electro-Optical Module (EOM). Fig. A.1 shows a scheme of the optics for PDH technique. The electric field of the modulated reflected laser beam is;

\[ E_r = E_0 e^{i\omega t} + E_h e^{i(\omega+\Omega)t} + E_l e^{i(\omega-\Omega)t}. \]

\( \omega \) is the frequency of the incident laser beam. \( \Omega \) is the modulation frequency. \( E_0 \) is the amplitude of the incident laser beam. \( E_h \) shows electric field of sideband of \( \omega + \Omega \). \( E_l \) shows electric field of sideband of \( \omega - \Omega \).

The intensity of the reflected laser beam is [66];

\[ I_r = |E_r|^2 = |E_0|^2 + |E_h|^2 + |E_l|^2 + 2\Re [E_0 E^*_h - E^*_0 E_l] \cos (\Omega t) + 2\Im [E_0 E^*_h - E^*_0 E_l] \sin (\Omega t) + (2\Omega \text{ terms}). \]

The reflected laser beam from the optical resonant cavity is detected by a photo detector which cuts DC component. The output of the photo detector, \( V_r \), is mixed with the modulation signal, \( V_m \sin \Omega t \). The mixed signal is;

\[ V_r V_m \sin (\Omega t) \propto 2\Im [E_0 E^*_h - E^*_0 E_l] \sin^2 (\Omega t) + (2\Omega \text{ terms}) \sin (\Omega t) \]

\[ \propto \Im [E_0 E^*_h - E^*_0 E_l] [1 - \cos (2\Omega t)] + (2\Omega \text{ terms}) \sin (\Omega t). \]

Here we cut vibrational component by the low pass filter. We take signal as;

\[ V_{PDH} \propto \Im [E_0 E^*_h - E^*_0 E_l]. \]

This signal is the error signal of the PDH which is calculated as Fig. A.2. Horizontal axis is the length of the optical resonant cavity. The blue
line is the error signal of the PDH. The red line is transmitted light of the optical resonant cavity. The error signal inverts around the peak of transmitted light. Then we can lock the resonance of the optical resonant cavity using the error signal of PDH technique.

![Diagram of PDH technique](image)

Figure A.1: A scheme of the optics for PDH technique.

![Graph of PDH technique](image)

Figure A.2: Horizontal axis is the length of the optical resonant cavity. Blue line shows the error signal of PDH technique. Red line shows transmitted light of the optical resonant cavity [10].
Appendix B

Confabulation of the feedback system

We tried to control the length of the optical resonant cavity by the configuration of the feedback system which is shown in Fig. B.1 before experiment of Chapter 3. In Fig. B.1, red box shows the laser oscillator. Blue box shows the optical resonant cavity. Red lines show transmission of the laser pulse. Black lines show electrical signal. In this scheme, the laser trigger signal inputs to the commercial feedback circuit. The commercial feedback circuit outputs a signal based on phase difference of the laser trigger and electron RF frequency. The signal of the phase detector is feedback to the laser oscillator.

We tried to control the length of the optical resonant cavity by the piezo actuator of the optical resonant cavity with the shoulder feedback technique [71]. The signal of the transmission intensity is feedback to the PID circuit. The output signal of the PID circuit inputs to the piezo actuator of the optical cavity. It is this scheme which is constructed by double closed loops as the loop for the optical resonant cavity and the oscillator cavity. In this system, the aim point of the shoulder feedback was not locked. Fig. B.2 shows transmission of the optical resonant cavity with this feedback system. In this system, the oscillator cavity refers RF signal. The optical resonant cavity refers the oscillator cavity. Though the length of the optical resonant cavity can not follow up the length of the oscillator cavity because the piezo actuator of the optical resonant cavity is moving slower than the piezo actuator of the laser oscillator.
Figure B.1: The simplistic feedback system.
Red box shows the laser oscillator. Blue box shows the optical resonant cavity. Red lines show signal based on laser pulse. Black lines show electrical signal.

Figure B.2: The transmission of the optical resonant cavity with the simplistic feedback system.
Yellow line shows the transmission intensity of the optical resonant cavity.
Appendix C

Control Theory [72]

C.1 Laplace transform

The Laplace transform and its properties are introduced important when we use about automatic control. Here we define \( f(t) \) by a function of time \( t \geq 0 \). When

\[
|f(t)| \leq ke^{at},
\]

is required with all of a natural number more than \( \delta_0 \), \( F(s) \) converges by transform as;

\[
F(s) = \mathcal{L}[f(t)] = \int_0^\infty f(t)e^{-st}dt,
\]

here \( s \) is a complex number and \( \Re[s] > \delta_0 \). This transform is the Laplace transform. While \( f(t) \) is calculated at \( t \geq 0 \) by;

\[
f(t) = \mathcal{L}^{-1}[F(s)] = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} F(s)e^{st}ds,
\]

when \( c > \delta_0 \). This transform is the inverse Laplace transform.

C.1.1 Properties of the Laplace transform

The Laplace transform requires some properties. The Laplace transform has linearity as;

\[
\mathcal{L}[af(t) + bg(t)] = aF(s) + bG(s),
\]

\[
\mathcal{L}^{-1}[aF(s) + bG(s)] = af(t) + bg(t).
\]

A differential and an integral of the Laplace transform is shown as;

\[
\mathcal{L} \left[ \frac{df(t)}{dt} \right] = sF(s) - f(0),
\]

\[
\mathcal{L}^{-1} \left[ \int f(t)dt \right] = \frac{F(s)}{s} + \frac{df(0)}{dt}.
\]
Therefore, the differential of the Laplace transform is shown multiplication with $s$. While, the integral of the Laplace transform is division by $s$.

When time goes on $\pm \Delta t$, the Laplace transform of $f(t)$ is;

$$\mathcal{L}[f(t \pm \Delta t)] = e^{\pm \Delta ts}F(s). \quad (C.1)$$

$t \to 0$ and $t \to \infty$ limitation of $f(t)$ are;

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s), \quad (C.2)$$

$$\lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s).$$

The Laplace transform used well are summarized in Table C.1.

<table>
<thead>
<tr>
<th>Function</th>
<th>Laplace transform</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impulse function</td>
<td>$\delta(t)$</td>
</tr>
<tr>
<td>Step function</td>
<td>$1$</td>
</tr>
<tr>
<td>Ramp function</td>
<td>$\frac{1}{s}$</td>
</tr>
<tr>
<td>Exponential function</td>
<td>$e^{-at}$</td>
</tr>
<tr>
<td>Sine function</td>
<td>$\frac{\omega}{s^2 + \omega^2}$</td>
</tr>
<tr>
<td>Cosine function</td>
<td>$\frac{s}{s^2 + \omega^2}$</td>
</tr>
</tbody>
</table>

Table C.1: The Laplace transform used well are summarized.

C.1.2 The Laplace transform of RC circuit

We show a utility of the Laplace transform with RC circuit shown as Fig. C.1.

![RC series circuit diagram]

Figure C.1: RC series circuit

Resistance value of a resister is $R \, [\Omega]$, capacitance value of a capacitor is $C \, [F]$. It is assumed that current intensity $I(t)$ flows to the circuit when two are connected with the series, and voltage $V_{in}(t)$ is applied to each end. Initial conditions are put $V_{in}(0) = 0$, $I(0) = 0$. The equation
of the RC series circuit is:

\[ V_{in}(t) = V_R + V_C = RI(t) + \frac{1}{C} \int I(t) dt. \]  \hspace{1cm} (C.3)

Eq. (C.3) transformed by the Laplace transform is:

\[ \mathcal{L}[V_{in}(t)] = \mathcal{L}[V_R] + \mathcal{L}[V_C] = \left( R + \frac{1}{Cs} \right) \mathcal{L}[I(t)]. \]

Here applied voltage of the resister, \( V_R \), is:

\[ \frac{\mathcal{L}[V_{in}(t)]}{\mathcal{L}[V_R]} = 1 + \frac{1}{RCs}, \]
\[ \mathcal{L}[V_R] = \frac{RCs}{1 + RCs} \mathcal{L}[V_{in}(t)]. \]  \hspace{1cm} (C.4)

C.1.2.1 RC series circuit applied DC voltage

We put applied voltage of the step function as:

\[ V_{in}(t) = \begin{cases} 
0 & (t < 0), \\
V & (t \geq 0).
\end{cases} \]

From Table. C.1, the step function transformed by the Laplace transform is \( 1/s \). \( RC \) is put \( \tau \) because \( RC \) has time dimension. Eq. (C.4) is written as:

\[ \mathcal{L}[V_R] = \frac{\tau s}{1 + \tau s} \frac{V}{s} = \frac{V}{s + \frac{1}{\tau}}. \]  \hspace{1cm} (C.5)

Eq. (C.5) is transformed by the inverse Laplace transform as:

\[ V_R = Ve^{-\frac{t}{\tau}} = RI(t). \]  \hspace{1cm} (C.6)

Fig. C.2 shows Eq. (C.6)

C.1.2.2 RC series circuit applied AC voltage

We put \( V_{in}(t) = V \sin \omega t \). In a similar way, applied voltage of the resister is:

\[ \mathcal{L}[V_R] = \frac{s}{s + \frac{1}{\tau}} \frac{\omega V}{s^2 + \omega^2} \]
\[ = V \frac{\omega \tau}{1 + \omega^2 \tau^2} \left( \frac{\omega}{s^2 + \omega^2} + \frac{s}{s^2 + \omega^2} - \frac{1}{s + \frac{1}{\tau}} \right). \]  \hspace{1cm} (C.7)

Eq. (C.7) is transformed by the inverse Laplace transform as:

\[ V_R = \frac{V \omega \tau}{1 + \omega^2 \tau^2} (\omega \tau \sin \omega t + \cos \omega t - e^{-\frac{t}{\tau}}) = RI(t). \]  \hspace{1cm} (C.8)
When $\omega = 1 \text{ rad/sec}$ and $\tau = 1 \text{ sec}$, Fig. C.2 shows Eq. C.8.

In this way, we can solve easily differential equations using the Laplace transform. In addition to easy solution, input-output characteristics and frequency characteristics can be shown by the Laplace transformed equation. There will be described in §C.2.

### C.1.3 Transfer function

#### C.1.3.1 Transfer function and first-order lag element

When a circuit is applied voltage, $V_{in}(t)$, the circuit output $V_{out}(t)$ shown as Fig. C.4.

$V_{out}(t)$ shows $V_C$ in §C.1 because Fig. C.4 is same as RC circuit in §C.1. Thus, $V_{out}(t)$ is:

$$V_{out}(t) = V_C = V_{in}(t) - V_R.$$
When the circuit is applied voltage, $V_{in}(t)$, the circuit output $V_{out}(t)$. It is transformed by the Laplace transform as;

$$
\mathcal{L}[V_{out}(t)] = \mathcal{L}[V_{in}(t)] - \mathcal{L}[V_R] = \left(1 - \frac{\tau s}{1 + \tau s}\right) \mathcal{L}[V_{in}(t)] = \frac{1}{1 + \tau s} \mathcal{L}[V_{in}(t)].
$$

When step function, $\mathcal{L}[V_{in}(t)] = V/s$, inputs to this circuit, $V_{out}(t)$ is;

$$
V_{out}(t) = V(1 - e^{-\frac{t}{\tau}}).
$$

Eq. (C.10) is shown as Fig. C.5. At $t = \tau$, $V_{out}/V = 1 - 1/e \simeq 0.63$. \(\tau\) is called time constant and is used a indication for a response speed of the system.

To the way of Eq. (C.4) and Eq. (C.9), when the Laplace transformed input function, $\text{INPUT}(s)$, and the Laplace transformed output function, $\text{OUTPUT}(s)$ are shown as;

$$
\text{OUTPUT}(s) = G(s)\text{INPUT}(s).
$$

$G(s)$ is called the transfer function.

Especially, transfer function,

$$
G(s) = \frac{b}{s + a}.
$$
is a first-order lag element. When the sine function like Eq. (C.7) inputs to Eq. (C.9), output is:

\[
\text{OUTPUT}(s) = \frac{\sin \omega t - \omega \tau \cos \omega t + \omega \tau e^{-\frac{t}{\tau}}}{1 + \omega^2 \tau^2} \text{INPUT}(s). \tag{C.11}
\]

Eq. (C.7) shows the input waveform and Eq. (C.11) as the output waveform. The phase of the output waveform is delay form the phase of the input waveform.

![Figure C.6: The input and the output waveform when the sine function like Eq. (C.7) inputs to Eq. (C.9).](image)

There is higher-order lag element. There will be described in § C.4.

C.1.3.2 Dead time component

In the trivial knowledge, “Those who talked over the telephone by the cellular phone were not able to duet mutually” had been introduced by the television program. It is necessary to listen to other party’s voice so that A-san and B-san may duet, and to speak also by the person in question according to other party’s voice. When A-san and B-san are singing very much in the short distance, time that the voice is transmitted is short. However, it is not possible to duet because it is long as the time lag of the communication cannot be disregarded by the cellular phone.

The time lag is called dead time. The Laplace transform of the voice of A-san is \( V_A(s) \), and the time lag of the cellular phone is assumed to be \( \Delta t \), \( V_B(s) \) is the Laplace transform of A-san’s voice which reaches the ear of B-san. From Eq. (C.11) \( V_B(s) \) is:

\[
V_B(s) = e^{-\Delta t s} V_A(s). \tag{C.12}
\]
Here, the other influences such as cutting of phone call frequency was removed.

When the time lag, $\Delta t$, is not decided accurately, the dead time component is combined with the first-order lag element as shown;

$$G(s) = \frac{Ke^{-\Delta ts}}{1 + \tau s} \quad (K \text{ is constant})$$

Harmony shifts easily when the time lag is long. It means that the controlled system becomes unstable easily when the dead time is long.

C.1.4 Block diagram

Many elements are combined when the control system is constructed. The transfer function of the I/O signal, the control and controlled system shown in figure is a block diagram.

The block diagram of Eq. C.9 is shown as Fig. C.7.

![Figure C.7: The block diagram of Eq. C.9.](image)

Fig. C.8 shows the equivalent conversion chart when the control elements are combined.

![Figure C.8: Equivalent transformation of block diagram.](image)

In Fig. C.8 (c), I/O signal are put to be $IN(s)$ and $OUT(s)$. The input of $G(s)$ is put $IN_G(s)$. $OUT(s)$ is;

$$IN_G(s) = IN(s) \mp H(s)OUT(s), \quad (C.13)$$
$$OUT(s) = G(s)IN_G(s). \quad (C.14)$$

From Eq. C.13 and Eq. C.14, $OUT(s)$ removed $IN_G(s)$ is;

$$OUT(s) = \frac{G(s)}{1 \pm G(s)H(s)}IN(s).$$
The control does the output operation by using output information is a feedback control. When $\pm$ of Eq. \ref{eq:C.13} is the minus, it is a negative feedback. When $\pm$ of Eq. \ref{eq:C.13} is the plus, it is a positive feedback.

C.2 Frequency response of the control element

C.2.1 Frequency response

When $V_{in} \sin(\omega t + \phi_{in})$ inputs to the transfer function, $G(s)$, the output is $V_{out} \sin(\omega t + \phi_{out})$. The I/O ratio, $V_{out}/V_{in}$, shows gain, and the change of the phase, $\phi_{out} - \phi_{in}$, is called a phase difference.

When we put $s = i\omega$, the transfer function, $G(s)$, is shown as;

$$G(i\omega) = \Re[G(i\omega)] + i\Im[G(i\omega)].$$

$G(i\omega)$ is a frequency transfer function. Here, $\Re[G(i\omega)]$ and $\Im[G(i\omega)]$ show real part and imaginary part, respectively. The absolute value and the variation angle of $G(i\omega)$ are;

$$|G(i\omega)| = \sqrt{\Re[G(i\omega)]^2 + \Im[G(i\omega)]^2}, \quad \text{(C.15)}$$

$$\text{arg} G(i\omega) = \tan^{-1}\frac{\Im[G(i\omega)]}{\Re[G(i\omega)]}. \quad \text{(C.16)}$$

The absolute value, Eq. \ref{eq:C15} is even as the gain of $G(i\omega)$. The variation angle, Eq. \ref{eq:C16} is even as the phase difference of $G(i\omega)$.

The frequency transfer function of the first-order lag element is;

$$G(i\omega) = \frac{1}{1 + i\omega\tau} = \frac{1 - i\omega\tau}{1 + \omega^2\tau^2},$$

from Eq. \ref{eq:C15} and Eq. \ref{eq:C16}, the gain and the phase difference is;

$$|G(i\omega)| = \frac{1}{\sqrt{1 + \omega^2\tau^2}},$$

$$\text{arg} G(i\omega) = -\tan^{-1}\omega\tau.$$  

Figure where the gain and the phase difference are shown for the angular frequency or the frequency is called Bode diagram. In general, $20\log |G(i\omega)|$ [dB] is used as the gain. Fig. \ref{fig:C.9} shows Bode diagram of the first-order lag element.

We can visually know the frequency response of the transfer function. From Fig. \ref{fig:C.9}, the gain of the first-order lag element decreases on the boundary of $\omega = 1/\tau$. While, the phase difference is $\pi/4$ rad at $\omega = 1/\tau$, and approaches asymptotically $\pi/2$ rad. The frequency, $f = \omega/2\pi = 1/2\pi\tau$, is cutoff frequency.
From Eq. \( C.12 \) the frequency transfer function of the dead time is;

\[
G(i\omega) = e^{-i\omega \Delta t} = \cos \omega \Delta t - i \sin \omega \Delta t,
\]

the gain and the phase difference are;

\[
20 \log |G(i\omega)| = 20 \log 1 = 0,
\]

\[
\arg G(i\omega) = -\tan^{-1} \tan \omega \Delta t = -\omega \Delta t.
\]

0 dB shows one time. While, the phase difference increases in proportion to \( \omega \).

C.2.2 Differentiating element and integration element

We show frequency response of the differentiation element and the integration element. The transfer function and frequency transfer function of the differentiation element are;

\[
G(s) = s, \quad (C.17)
\]

\[
G(i\omega) = i\omega.
\]

The gain and phase difference are;

\[
20 \log |G(i\omega)| = 20 \log \omega,
\]

\[
\arg G(i\omega) = \tan^{-1} \frac{\omega}{0} = \tan^{-1} \infty = \frac{\pi}{2}.
\]

The transfer function and frequency transfer function of the integration element are;

\[
G(s) = \frac{1}{s},
\]

\[
G(i\omega) = -\frac{i}{\omega}.
\]
The gain and phase difference are:

\[
20 \log |G(i\omega)| = 20 \log \frac{1}{\omega}
\]

\[
\arg G(i\omega) = -\tan^{-1} \frac{1}{\omega} = -\tan^{-1} \infty = -\frac{\pi}{2}
\]

The output signal is \(\omega\) times and advanced phase for \(\pi/2\) rad than the input signal of the differentiation element.

The output signal is \(1/\omega\) times and reversed phase for \(\pi/2\) rad than the input signal of the integration element.

In general, a practical differentiation element is used instead of Eq. C.17 as;

\[
G(s) = \frac{\tau s}{1 + \frac{\tau}{\alpha} s},
\]

here \(\alpha\) is arbitrary constant. \(V_R\) part, Eq. C.5, in the RC series circuit is the practical differentiation element. In Fig. C.3, the phase advances for \(\pi/2\) rad like the differentiation element.

When \(\alpha = 1\), the frequency transfer function of the practical differentiation element is;

\[
G(i\omega) = \frac{\omega^2 \tau^2 + i\omega \tau}{1 + \omega^2 \tau^2}.
\]

The gain and the phase difference are;

\[
20 \log |G(i\omega)| = 20 \log \frac{\omega \tau}{\sqrt{1 + \omega^2 \tau^2}}
\]

\[
\arg G(i\omega) = \tan^{-1} \frac{1}{\omega \tau}.
\]

Fig. C.10 shows Bode diagram of the differentiation element, the integration element and the practical differentiation element. Here, the time constant of the practical differentiation element is \(\tau = 1\). The black line is the gain of the differentiation element. The black dashed line is the phase difference of the differentiation element. The red line is the gain of the integration element. The red dashed line is the phase difference of the integration element. The green line is the gain of the practical differentiation element. The green dashed line is the phase difference of the practical differentiation element. Phases of the differentiation element and the practical differentiation element are not always late. Gains of the differentiation element and the practical differentiation element are corresponding at \(\omega < 1/\tau\).

Especially, the element that the gain rises on the high frequency side is called a High Pass Filter (HPF) like the differentiation element. The element that the gain rises on the low frequency side is called a Low Pass
Filter (LPF) like the integration element. Moreover, the element that the gain rises on only the specific frequency is the Band Pass Filter (BPF). The element that the gain drops on only the specific frequency is called the Band Elimination Filter (BEF) or the Notch Filter (NF).

For instance, voltage $V_R$ that applies in resistance becomes BPF when the RLC series circuit is used, and $V_L + V_C$ becomes NF. Each transfer function are:

\[
\text{BPF} : G(s) = \frac{R}{s^2 + \frac{R}{L}s + \frac{1}{LC}}.
\]

\[
\text{NF} : G(s) = \frac{s^2 + \frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}.
\]

When $R/L = 2$, $1/LC = 4$, each Bode diagram are shown Fig. C.11.

When each gain peak and each phase difference invert, the angular frequency becomes $\omega_0 = 1/\sqrt{LC} = 2$. $\omega_0$ is called resonance frequency.

C.3 PID control

We show temperature control of room for describing PID control.

This time, we want to control the temperature of a room which is the timbered house of 39 years old that poor postgraduate student, Miyosan, is using. The calorie, $Q = T(t)/R$, each unit time proportional to the room temperature, $T(t)$ [°C], runs away outside because the heat-retaining property of this room is bad. The stove that is the only heating equipment of this room can be adjusted the output power from 0 to
10 continuously by the nob. When the value of the adjustment nob is put $V(t)$, the output calorie each unit time of the stove is put $W_S = V(t)w$. When the thermal capacity, $C$, of this room is constant at any temperature, the time change of in the temperature $T(t)$ of this room is;

$$C \frac{dT(t)}{dt} = W_S - Q$$

$$= V(t)w - \frac{T(t)}{R}.$$  

It is Laplace transformed as;

$$CsT(s) = V(s)w - \frac{T(s)}{R},$$

$$T(s) = \frac{w}{s + \frac{1}{RC}} V(s).$$  \hspace{1cm} (C.18)

Where the temperature felt that Miyo-san is the best is put $T$, target temperature, and the block diagram is as shown in Fig. C.12. The room temperature is controlled by changing the output adjustment nob, $V(s)$, of the stove.

Figure C.11: Bode diagram of BPF and NF.

Figure C.12: Bode diagram of time change of the temperature in this room.
C.3. PID CONTROL

C.3.1 on/off control

One day, the room temperature is 0 °C when Miyo-san goes back into the room.

Because the best temperature for Miyo-san is 20 °C, Miyo-san adjusts the output of the stove to the maximum, ten, for heating rapidly the room. After a while, the room temperature exceeds 25 °C. Because Miyo-san has sweated a little, Miyo-san turns off the output 0, in a word, the stove is turned off. After a while, the room temperature falls below 15 °C. Miyo-san who feels cold adjusts the switch of the stove to the maximum ten again. Thus, Miyo-san repeats on/off of the stove. Where \( w = 100 \), \( C = 200 \) and \( R = 0.1 \), the changing of the room temperature is shown as Fig. C.13.

![Figure C.13: Change in room temperature when stove is done on/off.](image)

The control such as case example is called an on/off control. It is feedback because the output of the control system is decided from the temperature. An electric kotatsu controlled by on/off control for keeping constant temperatures in the kotatsu. If the range of the changing temperature is narrowed up to several degrees or the suitable temperature can be kept, Miyo-san is able to spend time more comfortably.

C.3.2 P control

The on/off control always changes the room temperature in the vicinity of the suitable temperature. Because Miyo-san hates curved thing, Miyo-san takes home an optional parts P for the stove somewhere. The output of the stove can be controlled by assuming the value proportional to the difference between the room temperature and the target temperature.
when parts P are used. The limitation is not the bound pair of $V(t)$ for the convenience of the calculation. The block diagram of the entire system is shown in Fig. C.14.

![Figure C.14: The block diagram of the entire system by P control.](image)

From Eq. C.18, the changing of the temperature is;

$$T(s) = \frac{\frac{w}{C}K_P}{1 + \frac{\frac{w}{C}K_P}{T_C}} s + \frac{1}{RC} T_C,$$

If the same initial room temperature, $C, w, R, K_P = 2$ are used, the temperature change is shown in Fig. C.15.

![Figure C.15: The temperature change by P control.](image)

The change of the temperature is slightly different to the target temperature of the target.

The difference between the target value and the current value is called deflection, and the deflection is called the steady-state deviation and an offset by this control. The control that uses the output proportional to deflection is called P (Proportional) operation or P control. $K_P$ is called a proportional gain.

The steady-state deviation remains without fail at P control. From Eq. C.18, the room temperature $T$ after the adequate time is;

$$\lim_{t \to \infty} T(t) = \lim_{s \to 0} sT(s) = \frac{K_P V RC}{1 + K_P V RC} T_C.$$
It shows that $T(t)$ doesn’t become $T_C$ eternity by using only P control. When the proportion gain is too large, the vicinity of the target value is settled to the steady-state deviation while greatly vibrating.

C.3.2.1 PI control

Miyo-san’s mind as a little steady-state deviation can be permitted is narrow, and Miyo-san takes optional parts I home further. The output of the stove is controlled by using the integration value of the difference between the room temperature and the preset temperature when parts I are used.

The block diagram of the entire system is shown in Fig. C.16 when Miyo-san connects part I as parallel to parts P. $\tau_I$ is time constants of parts I.

![Block Diagram](image)

Figure C.16: The block diagram of the room temperature with PI control.

The temperature change is:

$$T(s) = \frac{wC(K_P + \frac{1}{\tau_I}s)}{1 + \frac{wC(K_P + \frac{1}{\tau_I}s)}{s}} T_C.$$  

When the same initial room temperature, $C, w, R, K_P = 2$ and $\tau_I = 4$ are used, the temperature change is shown in Fig. C.17.

![Temperature Change](image)

Figure C.17: The temperature change with PI control.

The control output by using the integration of deflection is called I (Integral) control and I operation. When I control is used in this case
together with P control, it is called the PI control. If I control is used, the steady-state deviation is completely corresponding as:

$$\lim_{t \to \infty} T(t) = \lim_{s \to 0} sT(s) = T_C.$$  

When the gain of I element grows by reducing $\tau_I$, system approaches a set value early. However the faction becomes unstable by very small $\tau_I$. The temperature in Fig. C.17 passes the target value. A good control is small the overshoot and is settled to the target value early.

C.3.2.2 PID control

Miyo-san takes optional parts D home further because Miyo-san can not sleep at night by anxious even in a few overshoots of temperature. The output of the stove is controlled by using the differential value of the difference between the room temperature and the preset temperature when parts D are used. When parts PI are connected in parallel like the example, the block diagram of the system is shown in Fig. C.18. $\tau_D$ is time constants of parts D.

![Figure C.18: The block diagram of the room temperature with PID control.](image)

The temperature change is;

$$T(s) = \frac{\frac{C}{w}(K_P + \frac{1}{\tau_I} + \tau_D s)}{s + \frac{C}{w}(K_P + \frac{1}{\tau_I} + \tau_D s)} \frac{T_C}{s}$$

When the same initial room temperature, $C, w, R, K_P = 2$, $\tau_I = 4$ and $\tau_D = 0.5$ are used, the temperature change is shown in Fig. C.19.

The current temperature is not corresponding to an initial temperature at $t = 0$ because output of D parts is infinity with the step function. The control output by using the differentiation of deflection is called D (Differential) control and D operation. When D control is used in this case together with PI control, it is called the PID control. If D control is used, the system is stabilized, and the overshoot is suppressed small. The gain of D element grows by increasing $\tau_D$.

If the room temperature is controlled by the PID control, Miyo-san will be able to live comfortably. Because the PID control is understood easily, and is also analyzed easy. Therefore, PID control is used to have explained in more than 90 % of all control[74].
Fig. C.20 shows the change in the room temperature when the imperfect differentiation described in § C.2 is used instead of the differentiating element. The overshoot, about 0.5 °C, is suppressed compared with the PI control. The rise time is later than Fig. C.19 because the system is not used the complete differentiation. As a result, the gain of P and I is increased, and time response of the system can be improved.

C.4 Stability of control system

C.4.1 Characteristic equation

We put the control system of second-order lag element as;

$$G(s) = \frac{1}{s^2 - 0.2s + 0.01}.$$
When the system is looped in targeted value 1, the output is;

\[
\text{OUT}(s) = \frac{G(s)}{1 + G(s)} \frac{1}{s} = \frac{1}{s^2 - 0.2s + 1.01}. \tag{C.19}
\]

The time change of this output goes away from the target value, and emanates as shown in Fig. C.21.

![Figure C.21: The time change of Eq. C.19.](image)

Eq. C.19 is the Laplace transformed as;

\[
\text{OUT}(t) = \frac{1}{1.01} [1 - \sqrt{1.01} e^{0.1t} \sin t + \tan^{-1}(-10)]. \tag{C.20}
\]

Eq. C.20 emanates because the index part of \( e \) is positive.

The characteristic equation is used for determining the emanation or the attenuation. The characteristic equation is;

\[
1 + G(s) = 0.
\]

This is just the denominator of feedback loop of the element, \( G(s) \). The root of the the characteristic equation, the characteristic root, is contained the index part of \( e \), Therefore, when the characteristic root is negative, the system attenuates. The system emanates when the characteristic root is positive. The characteristic root of Eq. C.19 is \( s = 0.1 \pm i \), then we expect that the system emanates and, it is reasonable with Fig. C.21 and Eq. C.20. We can distinguish whether the feedback system emanates without calculating the inverse Laplace transform.

### C.4.2 Nyquist stability criterion

In § C.4, the characteristic equation was able to be solved easily, but the solution might be difficult. There is Nyquist plot as a method of judging stability without solving the characteristic equation.
We put a feedback loop, \( G(s)/(1+G(s)) \), like Eq. C.19. The frequency transfer function, \( G(i\omega) \), is plotted in a complex plane by \( \omega \) as parameter. If tracks are on right side from \(-1+0i\), the feedback system is stability. These is method of judging stability by Nyquist plot, Nyquist stability criterion.

For example, we put the system of first-order lag element as:

\[
G(s) = \frac{Ke^{-\Delta ts}}{\tau s + 1}. \tag{C.21}
\]

When \( K = 2, \Delta t = 1 \) and \( \tau = 1 \), \( G(i\omega) \) is plotted in the complex plane as shown Fig. C.22. In this case, the system is stability because tracks are on right side from \(-1+0i\).

![Nyquist plot of Eq. C.21 with K = 1, Δt = 1 and τ = 1.](image)

When \( K = 2, \Delta t = 5 \) and \( \tau = 2 \), \( G(i\omega) \) is plotted in the complex plane as shown Fig. C.23. In this case, tracks exceeds \(-1+0i\). When the phase difference of the output becomes \(-\pi\) with particular frequency, about \( \omega = 0.53 \) in this case, the gain is larger than 1 time. The output increases in exponential by the negative feedback loop.

The feedback system become to stabilize; when the gain grows more than 1, the phase difference only have to be smaller than \(-\pi\); when the phase difference is \(-\pi\), the gain is smaller than 1. Even if tracks keep slightly to the right from emanation point, the system becomes unstable by a turbulence etc. Thus, how away it is from \(-1+0i\) when the stability level of the feedback system is evaluated. A gain margin and a phase margin is used well by evaluating the stability level. The gain margin shows a inverse number of the gain where the phase difference is \(-\pi\). The phase margin shows the difference with \(-\pi\) where the gain is 1. For instance, Fig. C.24 shows the gain margin and the phase margin of Bode
Figure C.23: Nyquist plot of Eq. C.21 with $K = 2$, $\Delta t = 5$ and $\tau = 2$.

In Fig. C.24, the gain margin is about 8 dB, and the phase margin is about 70 degree. In general, the gain margin is required 1~3 dB and, the phase margin is required more than 20 degree with fixed command control. The gain margin is required 10~20 dB and, the phase margin is required 40~60 degree with follow-up control [75].

Fig. C.25 and Fig. C.26 show margins and Bode diagram of PI control and PID control in § C.3. Because the phase difference of PI control is $-40$ degree, the phase margin is 140 degree. There is not frequency where the phase difference becomes $-180$ degree. While there is not frequency where the gain becomes 0 dB and the phase difference becomes $-180$ degree with PID control. The phase difference is small when the gain is 0 dB. It shows that PID control is higher the stability level than
C.4. STABILITY OF CONTROL SYSTEM

PI control.

Figure C.25: Margins and Bode diagram of PI control.

Figure C.26: Margins and Bode diagram of PID control.
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